1. For each of the six situations below, use the right-hand rule to determine the direction of the magnetic field at P.
2. In each of the following four figures, a portion of the current carrying wire is boxed. For only this section of wire, draw the vector \( dl \) which describes the direction of this section. Also, draw the vector that points from this section to the point \( P \) (where you would like to determine the magnetic field) and label this \( r \). Mark the angle \( \theta \) that lies between these two vectors.

The Biot-Savart law describes the magnetic field created by a small section of current carrying wire at a particular location.

\[
d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{Id\ell \times \hat{r}}{r^2}
\]

By summing all of magnetic field contributions (due to all of the small sections of wire) we can then find the total magnetic field at our location. It is important to remember that in order to properly evaluate an integral we need all of the quantities to either be constant or written in terms of our integration variable. In order to have only one variable inside the integral we often need to rewrite \( \theta \), \( r \) or \( dl \). (Have you figured out where \( \theta \) comes in? We will want to write down the magnitude of the cross product as \( |d\ell \times \hat{r}| = dl \ r \sin \theta \).)
3. As always, setting up the integral is often more difficult than actually doing the integral. And, the physics is in setting up the integral. Let’s set up a few integrals. For each of the following configurations, write down expressions for (the magnitudes of) $dl, r, \sin \theta$, as well as identify the variable that will be integrated and the limits.

First, here is a sample.

The situation is an infinitely long wire, carrying current $I$ along the x axis. The point at which we’re interested in finding the field is located at $(0, h)$.

First, let’s locate a small section of the wire and label it $dl$. Our segment is at a distance $x$ away from the origin. We also draw the vector from this segment to $P$, $r$, and the angle $\theta$.

We now want to write the magnitude of $dl$.

$$dl = dx.$$  

The magnitude of $r$ is simply given by the length of the hypotenuse.

$$r = \sqrt{x^2 + h^2}$$

$\sin \theta$ is given by

$$\sin \theta = \frac{h}{r} = \frac{h}{\sqrt{h^2 + x^2}}$$

We want to integrate along the wire, so we are integrating the variable $x$ from $-\infty$ to $+\infty$.

Notice that $dl, r$ and $\sin \theta$ are written in terms of our variable and other constants. Assembling all of the parts, we find:

$$B = \frac{\mu_0 I}{4\pi} \int \frac{d\ell \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{\sin \theta d\ell}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{h dx}{(h^2 + x^2)^{3/2}}$$

Notice that we’re only finding the magnitude of the field (we added the absolute value symbols to the integral). To find the direction we will use the right hand rule.
Okay, now it’s time for you to try one.

First, here is a sample.

The situation is a line segment of length $L$ that is carrying current $I$ along the $x$ axis. The wire extends from $x=0$ to $x=L$. The point at which we’re interested in finding the field is located at $(0, h)$.

First, draw $dl$, $r$, and $\theta$ on the figure.

Write the magnitude of $dl$.

$dl =$

Now the magnitude of $r$.

$r =$

Now $\sin \theta$.

$\sin \theta =$

Variable of integration:

Limits of integration: from: to:
One more… The situation is a line segment of length L that is carrying current I along the x axis. The wire extends from x=0 to x=L. The point at which we’re interested in finding the field is located at (-d,h).

First, draw dl, r, and θ on the figure.

Write the magnitude of dl.

dl =

Now the magnitude of r.

r =

Now sin θ.

sin θ =

Variable of integration:

Limits of integration: from: to:

On the following page is another configuration of a current carrying wire. For this one, go ahead and solve the integral. One hint- since the wire is bent, treat it as three separate segments, two straight segments and one semicircle. Find the field due to each segment at P and then add the three to find the total field. (Be on the look out for “easy integrals”.)
4. Determine the magnetic field at point P. The wire carries current I and is bent into a semicircle of radius R. P lies at the center of the semicircle. The ends of the wire extend horizontally to a distance away from P that is much greater than R.
5. Each of the eight conductors carries 2.0A current into or out of the page. Four paths are indicated for the line integral $\oint \vec{B} \cdot d\vec{l}$. What is the value of the integral for each of the paths?

A: ___________________

B: ___________________

C: ___________________

D: ___________________

6. Describe the similarities between Ampere’s law in magnetism and Gauss’ law in electrostatics.