The Navy wants a new airplane launcher for their aircraft carriers and you are on the design team. The launcher is effectively a large spring that pushes the plane for the first 5 meters of the 20 meter long runway. During that same time, the plane’s jet engines supply a constant thrust of \(5.4 \times 10^4\) N for the entire length of the runway. The 2000 kg plane need to have a velocity of 45 m/s by the end of the runway. What should be the spring constant for the launcher?

The choice of initial and final moments let’s us use the information that we have to find what we want. We’ll use conservation of energy, where there will be kinetic and potential (spring) energies. (If we assume that the system is the plane and spring.) In addition, we’ll need to include the force of the engines. (Well,… we all know we are really taking about the force of the air on the plane as the engines push on the air, not the pane itself, right? Newton’s Third Law comes to the rescue.) This work supplied by this nonconservative force needs to be included. This additional energy means that the initial and final mechanical energies will not be equal; instead,

\[ U_s + W = K_f \]

We’ll solve for the spring constant, which will be the only unknown. I would expect it to be very large as such a spring must be very stiff.

- We’ll ignore any air resistance or friction.
- We’ll assume that the flight deck is horizontal; thus no change in the gravitational potential energy.
- We’ll assume that the engines are on, at the same value, the entire 20m.

\[ m = 2000\text{kg} \] (mass of the plane)
\[ f = 5.4 \times 10^4\text{N} \] (force on the plane due to the engines)
\[ \Delta x = 5\text{m} \] (distance over which the spring exerts a force)
\( d = 20 \text{m} \) (distance over which the engines are on)
\( v = 45 \text{ m/s} \) (plane’s final speed)
\( k \) (unknown spring constant)

The initial energy is solely in the form of spring potential energy:

\[ U_i = \frac{1}{2} k \Delta x^2 \]

The final energy is only in the form of kinetic energy.

\[ K = \frac{1}{2} m v^2 \]

The work of the constant, nonconservative force is given by:

\[ W = f d \]

We can now assemble all of the parts and solve for the spring constant:

\[ \frac{1}{2} k \Delta x^2 + fd = \frac{1}{2} mv^2 \]

\[ \Rightarrow k = \frac{mv^2 - 2fd}{\Delta x^2} \]

Substituting in the given values yields a spring constant of 75,600 N/m.

Is this reasonable? Well, I know it is a bigger number than a Slinky’s, which is around 1 N/m. Problem 7.17P of our book mentions that a truck’s springs are around \( 10^5 \) N/m. While ours is smaller, remember that the force on the plane will be quite large given the 5m compression.
After graduation you decide to join the circus. (It’s always been your dream.) Since you have taken physics courses, your supervisors give you the task of helping to design the human cannonball stunt. Your task is to design a stunt in which a man, who weighs 170 pounds, will be shot out of a cannon that is elevated 40° from the horizontal. The “cannon” is actually a 3-foot diameter tube that uses a stiff spring and a puff of smoke rather than an explosive to launch the man. The manual for the cannon states that the spring constant is 985 Newtons/meter. A motor compresses the spring until its free end is level with the bottom of the cannon tube, which is 5 feet above the ground. A small seat is attached to the free end of the spring for the man to sit on. When the spring is released, it extends 8 feet up the tube. Neither the seat nor the chair touch the sides of the 12-foot long tube. After a drum roll, the spring will be released and the stunt man will fly through the air with the appropriate sound effects and smoke. Currently, the circus is only traveling with an airbag which is 4-feet thick. You know that the airbag will exert an average retarding force of 2750 Newtons in all directions. You need to determine if this airbag is thick enough to stop the human cannonball safely.

While there is a lot of information in this description, the physics remains the same as the previous problem- the initial energy plus any work (by nonconservative forces) equals the final energy.

What is our system? Here it makes sense to look at the performer, spring and earth. (Notice that I did not include the airbag as it exerts a nonconservative force on the system.)

What are our initial and final points? There are many possibilities, but the easiest is to start the “experiment” when the spring is compressed, and the performer is not yet moving. The end is when the performer has come to rest on the airbag (and/or floor). With these “bookends” to the experiment, the initial speed and kinetic energy are both zero, as are the final quantities.

Next, we have to include any potential energies. Here there will be initial potential energy stored in the spring and none at the end. There will also be a change in gravitational potential energy (due to a difference in heights), but the exact numbers will depend on our choice of origin.

There are couple different routes one could take to decide if the airbag is sufficient:
• Let the performer’s final position be a variable. Then compare this result to the airbag’s thickness.
• Use the given airbag’s thickness and determine if the performer actually comes to rest by the time he falls the 4 feet. (Compute his final speed)
• Compute the change in total energy as well as the work from the airbag (with the given values). Then compare the two.

I’m sure there are other ways one could handle this problem. I’m going to use the first, not because it is better or even easier, it is just the method that I saw first. All of these methods, and possibly others, would be perfectly reasonable.
From the size of the airbag, I’m a bit skeptical that this stunt will work. From what I remember of circus performances, they typically use large nets, which give more than 4 feet. Also, when movie stunt persons do use airbags, I believe that they are thicker than 4 feet.

- I’m going to neglect air resistance.
- I’m going to assume that the spring is ideal and not deformed. (We can use the text’s equations for spring potential energy.)
- I’m going to assume that the force due to the airbag is constant. This greatly simplifies the airbag’s work calculations.
- Also, we’ll assume that the airbag is on the ground (which is flat).
- One subtle point that we haven’t had to mention before, is the assumption that the person is a point mass. Essentially we’re ignoring his height and width. By doing this, the work done by gravity is much easier to calculate. If we were to use his actual shape we would need to alter the gravitational work. (This gets into the idea of center of mass which we’ll discuss more this semester.) For example, we will say that he is located at the very bottom of the cannon, which isn’t quite correct (unless we assume that he is a point particle).

\[y_i = 5 \text{ ft} = 1.5 \text{ m} \text{ (performer’s initial position)}\]
\[y_f \text{ (performer’s final position)}\]

*** *y* > 0 would mean that the performer is stopped by the airbag, and *y* < 0 would mean that the performer wouldn’t come to rest until he falls through the floor. ***

\[h = 4 \text{ ft} = 1.2 \text{ m} \text{ (height of airbag)}\]
\[d = 8 \text{ ft} = 2.4 \text{ m} \text{ (length of extended spring)}\]
\[k = 985 \text{ N/m} \text{ (spring constant)}\]
\[f = 2750 \text{ N} \text{ (airbag’s force on the performer)}\]
\[m = 77 \text{ kg} \text{ (mass of the performer)} \text{ (found from his weight, 170 lbs)}\]

We’ve already stated that the initial and final kinetic energies are zero. Now we just need to combine the various potential energies and airbag work.
\[ W + U_g + U_a = U_{gf} \]

The spring’s potential energy is given by:

\[ I = \frac{1}{2} k d^2 \]

We can use the initial and final positions to determine the gravitational potential energy at the two different moments. (Remember, we’re letting the final position be a variable which we can then compare to the airbag’s thickness.)

\[ U_{gi} = m g y_i \]
\[ U_{gf} = m g y_f \]

On important note about the gravitational energy. It is important to include the distance the performer falls while in contact with the airbag. Gravity never turns off.

Since we’re assuming that the airbag exerts a constant force, we can say that the work it does is:

\[ W = f (y_f - h) \]

Notice that this work is negative (\( y_f < h \)) - the airbag slows down the performer.

Now we simply sum up the energies, and solve for \( y_f \). \( y_f > 0 \) would mean that the performer is stopped by the airbag, and \( y_f < 0 \) would mean that the performer wouldn’t come to rest until he falls through the floor.

\[ \frac{1}{2} k d^2 + m g y_i + f (y_f - h) = m g y_f \]

We can now insert the values of \( k, d, m, g, y_i, h, \) and \( f \) to find \( y_f \).

After some math we find that \( y_f = -0.3 \text{ m} = 1 \text{ ft} \) below the ground.

The air bag is not thick enough. It would need to extend below the ground to safely stop the performer.

Overall, what we’ve done here seems reasonable, and it does fit with what’s seen at circuses & stunts.