Assignment #7, Physics 321  
Due Date: Monday 30 November 2009

1. (a) Build the one-dimensional QHO wavefunctions for $n = 2, 3, 4$ using the raising and lowering operators.

(b) Verify that your functions are orthogonal and normalized.

(c) Plot and compare these to the “pre-defined” QHO wavefunctions that come from the solution to the Schrödinger equation (the one with the Hermite polynomials!).

2. Consider a time-dependent QHO that consists of an equal mixture of two states, $\Phi(x, t) = \frac{1}{\sqrt{2}} (\phi_0(x, t) + \phi_4(x, t))$, using your answers from question 1.

(a) Write out the wavefunction $\Phi(x, t)$, including the explicit form of the energies in the time-dependent terms.

(b) Derive the form of the probability density $\rho(x, t) = \Phi^*(x, t)\Phi(x, t)$, and show that its normalization is time-independent.

(c) Study and discuss the motion of the QHO over the interval $t = 0$ to $t = 10$. You can use the animate function in Maple if you want (type ?animate at the command line to see how you can use it, or alternative check out the Maple worksheet from 12 October on the website).

3. In class, we discussed the functional form of the solutions to the quantum harmonic oscillator potential. That is, we discussed the form of the eigenvectors cast into the $|x\rangle$ representation, $\phi_n(x) = \langle x|n\rangle$. Recall that the beauty of quantum mechanics is that we can do all the mathematical manipulation in Hilbert space by describing the eigenfunctions as vectors, and the operators as matrices.

For this exercise, let’s consider the eigenvectors $|0\rangle, |1\rangle, |2\rangle, \ldots$ in the usual vector form, e.g. $|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$, and so on.

(a) Write the matrix form of the raising and lowering operators, $a^\dagger$ and $a$, based on how each operator acts on a given state $|n\rangle$ (e.g. $a|0\rangle = 0$, $a^\dagger|0\rangle = |1\rangle$, etc...).

(b) Write the matrix form of the QHO Hamiltonian using your matrices from part (a), and verify that you get the right energies $H|n\rangle = E_n|n\rangle$.

(c) Show that the number operator $N = a^\dagger a$ is Hermitian using your derived matrices.