The evaluation of integrals does not end in Calculus courses

Victor H. Moll
Department of Mathematics
Tulane University
New Orleans
Louisiana

There is a well established rich connection between the evaluation of definite integrals and special functions. For instance, everybody knows that

$$\int_{0}^{\infty} e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2}$$

and that

$$\int_{0}^{1} e^{-x^2} \, dx = \frac{\sqrt{\pi}}{2} \text{Erf}(1).$$

In this talk I will present some new connections between the evaluation of definite integrals and dynamical systems, number theory and combinatorics. Some open questions will be discussed. For example the values of

$$\int_{0}^{1} \ln(x - 1)! \, dx$$

and

$$\int_{0}^{1} \ln^2(x - 1)! \, dx$$

will be discussed. The evaluation of

$$\int_{0}^{1} \ln^3(x - 1)! \, dx$$

is an open problem.