Supply and Demand Functions

In these notes, I will briefly discuss supply and demand functions and explain them properly.

**Demand Function:** The demand function is a function in which price is written as dependent on demand. However, in the words we use to talk about demand functions, we discuss demand as dependent on price. (This is what caused me to make the mistakes.) So our thinking is that as price goes up, demand goes down, and as price goes down, demand goes up. The notion being that nobody would buy a $100 hamburger. For graphing, we put the price on the $y$-axis and the demand on the $x$-axis. In our hypothetical hamburger example, a price of $100 would yield a demand of 0 hamburgers/day. On the other hand, if a hamburger costs 10 cents, you would expect a very large demand, say 1000/day. This means that we would see a graph with points $(0, 100)$ (the first coordinate is the number of units demanded, the second coordinate is the price) and $(1000, 0.10)$ (for a demand of 1000 and a price of $0.10). Assuming the demand function is linear, this is a line with a negative slope. To write the equation of the graph, we find the slope:

\[ m = \frac{0.10 - 100}{1000 - 0} = -0.1001. \]

Since the $y$-intercept is 100, the formula for the demand function is

\[ P(x) = -0.1001x + 100. \]

**Supply Function:** The supply function works the same as the demand function with units on the $x$-axis and price on the $y$-axis. Again, however, we talk about the process in terms of the price being the independent variable. In this case, we have that when the price is large, the supply is larger. (Lots of people (lets say we would get 5000/week) want to produce hamburgers if they get paid $100 each for them, but when the price is small, the supply is small (if a hamburger costs $1 to make, nobody would make any if they only got paid $1 for it). In this case, we see that the supply is larger when the price is larger, so the slope should be positive. If we wanted to plot the points this time, we would have the points $(5000, 100)$ and $(0, 1)$. We need a slope

\[ m = \frac{100 - 1}{5000 - 0} = 0.0198. \]
Using the point-slope formula for a line, we have the following chain:

\[ y - 0 = 0.0198(x - 1) \]
\[ y = 0.0198x - 0.0198 \]

Since \( y \) is the price \( P(x) \), we have the function

\[ P(x) = 0.0198x - 0.0198. \]

**Exercise 21.** The quantity demanded of a certain brand of DVD player is 3000/wk when the unit price is $485. For each decrease in unit price of $20 below $485, the quantity demanded increases by 250 units. The suppliers will not market any DVD players if the unit price is $300 or lower. But at a unit price of $525, they are willing to make available 2500 units in the market. The supply equation is also known to be linear.

1. Find the demand equation.
2. Find the supply equation.
3. Find the equilibrium quantity and price.

**Solution:**

a. To find the demand equation, there are two methods. One is to decide on what the slope of the function should be based on the second sentence. The other is to use the information given to find a second point. Essentially, knowing that a unit price decrease of 20 caused demand to increase by 250 tells us that the slope (which should be negative) is \( m = \frac{-20}{250} \).

Using the point-slope equation, we have

\[ y - 485 = \frac{-20}{250}(x - 3000) \]
\[ y = \frac{-2}{25}x + 240 + 485 \]
\[ y = \frac{-2}{25}x + 725. \]

So the demand function is (replacing \( y \) with \( P(x) \)). \( P(x) = \frac{-2}{25}x + 725. \)

b. For the supply equation, we have been given two points \((0, 300)\) (0 units produced when price is 300 dollars) and \((2500, 525)\). First we calculate the slope:

\[ m = \frac{525 - 300}{2500 - 0} = \frac{225}{2500} = \frac{9}{100}. \]
Now that we have the slope and the point \((0, 300)\), we have the \(y\)-intercept is 300. Thus the equation is

\[ y = \frac{9}{100}x + 300. \]

Replacing \(y\) with \(P(x)\) (the price), the supply function is

\[ P(x) = \frac{9}{100}x + 300. \]

c. To find the equilibrium, we set the two functions equal to each other. To get

\[ -\frac{2}{25}x + 725 = \frac{9}{100}x + 300. \]

Adding \(\frac{2}{25}x\) to both sides and subtracting 300 from both sides, we arrive at

\[ 425 = \left(\frac{2}{25} + \frac{9}{100}\right)x = \frac{17}{100}x \]

Multiplying both sides by \(\frac{100}{17}\), we have

\[ 2500 = x. \]

Consequently, the number of units for equilibrium is 2500/wk. The easy way to find the price is to notice that the problem told us that one of the points on the line had coordinates \((2500, 525)\). Another option is to substitute 2500 in for \(x\) in either of the equations.