Study Guide for Final Exam

The final will be a 120 minute test that counts for 40% of your grade. It will consist of definitions, basic mechanical problems, more complex problems requiring arguments including counting problems, and one or two proofs. Any homework type problem is fair game, any definition that we have used is fair game, and any named theorem/lemma/corollary is fair game. Moreover, there will likely be one or two problems that you haven’t seen before, but have all the tools to solve.

The exam will come in two sections, one in which the problems will be basic definitions, statements of theorems, identifying basic generating functions, etc. You will be given a formula sheet of ordinary and exponential generating functions for the second section. You may also use a calculator on the second section.

The list below is not meant to be exhaustive, merely to help you identify important topics from class.

1. Generating Functions

- You should know the definition of both the ordinary and exponential generating function for a sequence \( \{a_n\} \).
- You should know the exponential and ordinary generating function for \( a_n = r^n \).
- You should know the ordinary generating function for the sequence \( a_n = \begin{cases} b^n & r = nk \text{ for some } n \\ 0 & \text{otherwise} \end{cases} \)
- You should know the binomial theorem.
- You should know the definition of the characteristic polynomial for a recurrence relation.
- Be able to set up solutions to counting problems using generating functions (both ordinary and exponential) for problems.
- Be able to find the coefficients of terms in generating functions.
- You should know how to set up a recurrence relation and solve it using either the characteristic polynomial or generating functions.
- Sample problems on using generating functions to solve recurrence relations. Section 7.5: 1, 3, 4, 7, 8
2. Enumeration

- You should know the addition and the multiplication principles.
- You should be able to solve balls and box type problems both directly and through generating functions as appropriate.
- You should know the elementary counting methods/principles ($\binom{n}{k}$, $P(n, k)$, etc.)
- You should be able to do more complex enumeration problems with explanations for why the approach gives the correct answer.
- You should be able to do proofs involving binomial identities.
- You should have a working knowledge of block-walking problems and Pascal’s triangle.

3. Graph Theory:

- Definitions: The definition of a graph, a vertex, an edge, the degree of a vertex, a bipartite graph, walk, path, circuit, tr and cycle. In addition, you should know the definition of a planar graph, a Graph Coloring, a network, and a flow. You should know the definition of an isomorphism of graphs.
- Theorems, Corollaries, and Lemmas: You should know the statement of any named theorem (Handshake lemma, max-flow min-cut theorem, etc.) You should also know the main results of the important un-named theorems for use in problems and/or proofs.
- Induction: You should be able to do a basic induction proof.
- You should be able to determine whether two graphs are isomorphic, and to justify your claim.
- You should be able to use and identify graph models for problems, and you should be able to use the model to help you problem solve (in particular the sum of the degrees theorem should be useful here).
- You should be able to determine whether a graph is planar, and find a $K_{3,3}$ or $K_{5}$ configuration if it is non-planar.
- You should be able to do basic problems about graph coloring.
- You should be able to apply Euler’s Formula.
• You should be able to apply the flow algorithm to augment a flow in a network. (For example problems 4.3: 1, 2, 3)