1. While watching the local TV news show, you see a report about ground water contamination and how it effects farms which get their water from wells. For dramatic effect, the reporter stands next to an old style well, which still works by lowering a bucket at the end of a rope into a deep hole in the ground to get water. At the top of the well a single vertical pulley is mounted to help raise and lower the bucket. The thin rope passes over the large pulley which is essentially a heavy steel ring supported by light spokes. To demonstrate the depth of the well, the reporter completely wraps the rope around the pulley and suspends the bucket from one end. She then releases the bucket, at rest near the pulley, and it descends to the bottom of the well unwinding the rope from the pulley as it falls. It takes 2.5 seconds. She doesn't tell you the depth of the well so you decide to calculate it. You estimate that the pulley has the same mass of the bucket and assume that the mass of the rope and any friction can be neglected.

As a quick estimate, let’s first assume a freely falling bucket. How deep would a well be if it takes the bucket 2.5 sec to reach the bottom?

\[ \Delta y = \frac{1}{2} g t^2 = \frac{1}{2} \left(9.8 \text{ m/s}^2 \right) (2.5 \text{ s})^2 = 30.6 \text{ m} \]

This gives us the upper limit of the well depth. Our well will be less deep as the bucket’s acceleration will be less than 9.8 m/s\(^2\). How do we know? Well, this bucket is connected to a rope, which is wrapped around a pulley. It will take energy to spin the pulley, which is not in our rough estimate. Nonetheless, 30m is a good starting estimate.

To tackle this problem (with the pulley) we’ll need to employ Newton’s Second law and kinematic equations. (Why this route, instead of energy methods? We are given a time and time is easily related to acceleration via a kinematic equation. It is difficult, although not impossible, to connect time to energy.)

- We’ll assume that there is no friction in the pulley’s axle, but there is sufficient friction between the rope and pulley such that the rope does not slip.
- We’ll approximate the pulley by a simple ring and ignore the “light spokes.”
- We’ll go with the equal mass assumption.

First let’s draw FBD for each object.
We can apply the linear version of Newton’s Second Law to the bucket.

\[ T - mg = -ma \]

Solve this for the rope’s tension.

\[ T = m(g - a) \]

We can also use Newton’s Second Law (rotational version) on the pulley, with the pivot at the center of the pulley.

\[ TR = I\alpha \]

Now we’ll make use of the pulley’s shape and substitute in the moment of inertia for a simple hoop.

\[ TR = mR^2\alpha \]

Next, make use of the no slip condition to relate the angular acceleration of the pulley to the linear acceleration of the rope & bucket.

\[ TR = mR^2\left(\frac{a}{R}\right) \Rightarrow T = ma \]

This can be substituted into the equation for the bucket, so we can solve for acceleration.

\[ ma = m(g - a) \Rightarrow a = \frac{g}{2} \]

Now we’re ready to return to the same kinematic equation that we used in our rough estimate to find the distance that the bucket fell during 2.5 seconds.

\[ \Delta y = \frac{1}{2}\left(\frac{g}{2}\right)\Delta t^2 = \frac{1}{2}\left(4.9\text{ m/s}^2\right)(2.5\text{ s})^2 = 15.3\text{ m} \]

So, our initial estimate was a factor of two too large. This illustrates the importance of considering rotational motion along with linear motion.
2. On the weekend you find yourself engaged in serious physics experiments- bowling. You will throw the ball straight down the alley. When it starts, its center of mass will have a speed of 5m/s and it is sliding without rotating. You guess that the coefficient of friction between the ball and the floor is 0.1. Before you actually throw the ball, you decide to determine how far the ball will go down the alley before it starts rolling without slipping.

First, what is going on qualitatively? Why does the ball spin when it is initially released with no rotation? To cause a rotation we, of course, need an applied torque to the body, which comes from the friction between the floor and ball in our problem. When we start the ball spinning will we loose some translational kinetic energy. Now we have rotational kinetic energy (1/2 I \omega^2), and we have lost some energy (if the ball is our “system”) due to friction- heating up the lane. This means that the center of mass speed will decrease until the ball is rolling. Once rolling, there is no longer any kinetic friction or torque on the ball and v = \omega R. We can sketch out the center of mass speed and angular speed as functions of time.

![Graph of V vs time]

![Graph of \omega vs time]

Now we want to tackle the problem in a more quantitative manner; specifically, when does the ball begin to roll? From this we will be able to determine the distance the ball has traveled. What do we expect as a distance? If the numbers are realistic we should match our bowling experience (either in person or watching on ESPN). Bowling balls don’t immediately begin to roll- this depends a lot on the bowler, of course. We might expect something around a couple meters.

- \nu_0= 5m/s (initial speed of the ball)
- Let’s approximate the bowling ball as a uniform sphere with mass M, radius R
- \mu_k= 0.1 (coefficient of friction (kinetic) which is independent of position on the track (a well maintained alley))
• The previous assumption will imply that the force and torque are constant along the alley.
• Let \( t` \) denote the time at which rolling begins.

First, let’s find the angular and linear accelerations (notice that \( \alpha > 0 \) and \( a < 0 \)).

For a uniform, solid sphere, the moment of inertia can be looked up in the text:

\[
I = \frac{2}{5} M R^2
\]

The only torque we found in the first part of the problem was due to the friction, so

\[
\tau = RF = R(\mu_k M g)
\]

Recalling that \( \tau = I \alpha \), we find that:

\[
\alpha = \frac{5 \mu_k g}{2R}
\]

Now for the center of mass acceleration. The only horizontal force on the ball is the frictional force, \( F = -\mu_k M g \). With \( F = M a \), we find that:

\[
a = -\mu_k g
\]

As we discussed in class, when the ball is rolling we have \( v = \omega R \). Since both \( a \) and \( \alpha \) are constant in time (until the ball starts rolling), we can write the linear and angular speeds in terms of the accelerations.

\[
v = v_O + a t \\
\omega = \omega_O + \alpha t
\]

Here \( \omega_O = 0 \).

Now we just need to substitute the acceleration expressions into the speed equations and then use \( v = \omega R \).

\[
v = \omega R \\
v_O - \mu_k g t` = R \left( \frac{5 \mu_k g t`}{2R} \right)
\]

Notice that this is only true when \( t = t` \).

So, ...

\[
v_O = 3.5 \mu_k g t`
\]
No we can find the time at which the ball begins to roll without slipping, $t^* = 1.46$ sec. From this we can find the distance which the ball travels before it begins to roll. This is done using kinematic equations.

$$
\Delta x = v_0 t^* + \frac{1}{2} a t^{*2} = 5(1.46) - \frac{1}{2}(0.98)(1.46)^2 = 6.3 \text{ meters}
$$

This is a bit longer than my estimate, most likely due to a rather large initial speed in this problem, but certainly still within a reasonable range.