

Physics 274 In-Class Worksheet Solutions
29 Aug 2011

Orders of Magnitude and Fermi Problems

1. *How many gallons of gas are burned in Los Angeles each day?*

The population of greater Los Angeles is in the range of 10,000,000 people, or 10^7 . We'll assume that, on a given day, most of the are driving somewhere. Most will probably be commuting to work, anywhere between 10-100 miles round-trip. Others will be driving further. So, we'll assume that on average, people drive about 10 miles per day (since in reality not all people drive). This means that all the cars in LA travel $10 \times 10^7 = 10^8$ combined miles! The average gas consumption for cars ranges between 10 mpg (SUVs and large trucks) to above 50 mpg (EVs, hybrids and motorcycles). Most people don't have hybrids or motorcycles, though, so we can reasonably say that the average person gets 10 mpg, which means that a gallon of gas is burned for every 10 miles driven. So, if there are a total of 10^8 driven miles, the amount of gas burned must be $\boxed{10^7 \text{ gallons}}$ – ten *million* gallons per day! At \$3.00/gallon, that's \$30 million dollars burned each day!

2. *Estimate the weight of coffee beans used in the United States per day.*

Generally, it takes two tablespoons of coffee to make one cup. Let's estimate that this is about $0.1 = 10^{-1}$ lbs of coffee. People drink on the order of 1 cup per day, and there are roughly 100,000,000 = 10^8 people in the US. That means that, each day, a total of $\boxed{10^{-1} \times 10^8 = 10^7}$ pounds of coffee is consumed. There are actually 300,000,000 people, but they don't all drink coffee. So, this order-of-magnitude estimate (1/3 of the population) is probably close to accurate.

3. *How many grains of sand are there on the Santa Monica beach?*

The Santa Monica beach is about 10 km=10,000 m= 10^4 m long and 100 m wide. Let's say the sand *also* goes down about 10 m. Each grain of sand is bigger than a human hair (10^{-5} m) but much smaller than a fly (10^{-2} m), so let's say that it's about 10^{-4} m on a side. That means that 10^4 grains of sand lined up end-to-end would stretch 1 meter. So, the beach is actually 10^4 grains/meter $\times 10^4$ meters = 10^8 grains of sand long, 10^6 grains of sand wide, and 10^5 grains deep. So, there are approximately $\boxed{10^8 \times 10^6 \times 10^5 = 10^{19}}$ grains of sand on the beach.

We could also have said the the volume of sand on the beach is $10^4 \times 10^2 \times 10^1 = 10^7 \text{ m}^3$, and that the volume of each grain is $10^{-4} \times 10^{-4} \times 10^{-4} = 10^{-12} \text{ m}^3$. That means that the beach contains $\frac{10^7}{10^{-12}} = 10^{19}$ grains, which is the same answer.

4. *People are mostly made up of water. A water molecule weighs on the order of 10^{-26} kg. Estimate how many water molecules there are in the average human body, which is 90% water.*

The average person has a mass of about 100 kg. We can assume that humans are pretty much entirely made up of water, so all 100 kg = 10^2 kg is water. If each molecule has a mass of 10^{-26} kg, then each person is made up of about $\boxed{10^2 / 10^{-26} = 10^{28}}$ molecules of water!