Assignment #4 solutions

Physics 274

1. (a) To calculate the 5-psi radius, we need to use the graph to figure out the equivalent distance for the 1-kt explosion. So, we first need to use cube-root scaling to determine the height-of-burst for the 1-kt reference. In each case, this will be calculated by the formula

$$H_{1\text{ kt}} = \frac{H_{\text{Yield kt}}}{\sqrt[3]{\text{Yield}}}$$

where "Yield" is the yield of the bomb in question. Thus, in each case, the 1-kt scaled height-of-burst is:

Teapot Tesla: $H_{1\text{ kt}} = \frac{300}{\sqrt[3]{8}} = \frac{300}{2} = 150$ feet
Plumbob Hood: $H_{1\text{ kt}} = \frac{1500}{\sqrt[3]{74}} = \frac{1500}{5} = 300$ feet
Redwing Dakota: $H_{1\text{ kt}} = \frac{0}{\sqrt[3]{1000}} = 0$ feet

These are the heights at which a 1-kt bomb must be detonated to produce the same overpressure effects as the bombs in question. Note that since Redwing Dakota was a surface burst (0 feet), we don't need to scale the height-of-burst.

We can now use these values to read the 5-psi distance for the 1-kt burst from the overpressure graph. Once we know these, we need to scale the distances back to their actual value, according to the cube-root scaling relationship

$$d_{\text{Yield kt}} = d_{1\text{ kt}} \times \sqrt[3]{\text{Yield}}$$

This gives:

Teapot Tesla: 1-kt at 150 feet has 5-psi at about 1550 feet, so $d_{8\text{ kt}} = (1550) \times \sqrt[3]{8} = 3100$ feet

Plumbob Hood: 1-kt at 300 feet also has 5-psi at about 1750 feet, say, so $d_{15\text{ kt}} = (1750) \times \sqrt[3]{15} = 8750$ feet

Redwing Dakota: 1-kt surface burst has 5-psi at 1500 feet, so $d_{1000\text{ kt}} = (1500) \times \sqrt[3]{1000} = 15,000$ feet

Since there are 5280 feet in a mile, these distances are:

Teapot Tesla: $\boxed{d_{8\text{ kt}} = \frac{3100}{5280} = 0.59 \text{ miles}}$
Plumbob Hood: $\boxed{d_{15\text{ kt}} = \frac{8750}{5280} = 1.66 \text{ miles}}$

Redwing Dakota: $\boxed{d_{1000\text{ kt}} = \frac{15{,}000}{5280} = 2.84 \text{ miles}}$

2. The blast area for each is the area of the circle whose radius is the blast radius. So:

   Teapot Tesla: $\boxed{A_{\text{blast}} = \pi \times (0.59)^2 = 1.09 \text{ square miles}}$

   Plumbob Hood: $\boxed{A_{\text{blast}} = \pi \times (1.66)^2 = 8.66 \text{ sq-mi}}$

   Redwing Dakota: $\boxed{A_{\text{blast}} = \pi \times (2.84)^2 = 25.4 \text{ sq-mi}}$

3. To determine the burn radius, we need to use the thermal exposure graph to get the slant range for the appropriate exposure value. Both Teapot Tesla and Plumbob are "low-yield", so this value will be about 5-6-cal/cm^2. Redwing Dakota is a megaton-range weapon, so it should be a bit higher (about 8-cal/cm^2). For each bomb, the slant ranges are:

   Teapot Tesla: Slant range: $s \approx 1.3$ miles

   Plumbob Hood: Slant range: $s \approx 3.0$ miles

   Redwing Dakota: Slant range: $s \approx 7$ miles

   This is the hypothenuse of a triangle whose sides are the blast height (H) and the burn radius (R). So, to determine R, we use the Pythagorean formula $R\ sqrts^2 - H^2$, where we have to be sure to put H in miles:

   Teapot Tesla: $\boxed{R = \sqrt{(1.3)^2 - (0.06)^2} = 1.3 \text{ miles}}$

   Plumbob Hood: $\boxed{R = \sqrt{(3.0)^2 - (0.28)^2} \approx 3.0 \text{ miles}}$

   Redwing Dakota: $\boxed{R = \sqrt{(7)^2 - 0^2} = 7 \text{ miles}}$

   So, in all three cases, the burn radius is effectively the same as the slant distance!

4. The burn area is the area of the circle whose radius is the burn radius:

   Teapot Tesla: $\boxed{A_{\text{burn}} = \pi \times (1.3)^2 = 5.3 \text{ sq-miles}}$

   Plumbob Hood: $\boxed{A_{\text{burn}} = \pi \times (3.0)^2 = 28.3 \text{ sq-miles}}$

   Redwing Dakota: $\boxed{A_{\text{burn}} = \pi \times (7)^2 = 154 \text{ miles}}$

5. The casualty estimates are determined from the blast area, assuming that everyone inside that circle is a casualty. So, we use the formula:

$$\text{Casualties} = (\text{Pop. density}) \times (\text{Blast area})$$

Since the population density of LA is 7877 people per square mile, this implies:

Teapot Tesla: $\boxed{\text{Casualties} = 7877 \times 1.09 = 8586 \text{ people}}$

Plumbob Hood: $\boxed{\text{Casualties} = 7877 \times 28.3 = 222,919 \text{ people}}$

Redwing Dakota: $\boxed{\text{Casualties} = 7877 \times 154 = 1,213,058 \text{ people}}$