Assignment #6
Physics 322
Due before you get your exam!

1. Derive an expression the first-order *relativistic* corrections to the hydrogen wavefunction, using only the eigenfunctions from the 14 April worksheet. Recall that the unperturbed energies are the Bohr energies, $E_n^0 = -\frac{\mathcal{E}_0}{n^2}$.

2. A perturbation $W = \alpha\, \delta(x - \frac{a}{2})$ is placed in the middle of an infinite potential well that spans $0 \leq x \leq a$.

   (a) Find the first-order perturbation to the n-th energy level, E_n^1.

   (b) Determine the conditions in which this perturbation is non-zero.

   (c) Calculated the first three terms in the first-order wavefunction correction $\langle x|\psi_1^1\rangle = \psi_1^1(x)$.

   Note: in case you've never see it before, the *delta function* $\delta(x - a)$ describes a distribution that is concentrated at only one point. When you integrate it with another function, you get

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)\; dx = f(a)$$

3. The following question deals with the Klein-Gordon equation, and applying our "Think Different!" philosophy. Enjoy!

   (a) Suppose a solution to the KG equation is $\phi = e^{-i(Et-px)}$. Write down the equivalent negative energy solution, and describe what it represents (as we did in class).

   (b) For your solution in (a), what does ϕ^* represent?

   (c) Consider again the KG equation for a charged particle interacting with an electromagnetic field:

$$\left[(\partial_\mu + iqA_\mu)(\partial^\mu + iqA^\mu) + m^2\right]\phi = 0$$

   where ∂_μ is the four-dimensional derivative (gradient) operator and A_μ is the vector potential of the EM field. Take the complex-conjugate of this equation, so that you have a new equation for ϕ^*. Use this equation to further describe the difference between ϕ and ϕ^* (that is, what else does this equation tell you about antiparticles?).