

## Assignment #4

Physics 322

Due: Wednesday 24 March 2010

Please answer all questions, and turn in your solutions as a hard copy. If you use Maple, you may turn in your Maple output, so long as it is well-documented (not just Maple code and output). All questions are of equal value.

1. A system of two coupled spin-1/2 particles at time  $t = 0$  is in the state

$$|\psi(0)\rangle = \frac{1}{2}|++\rangle + \frac{1}{2}|+-\rangle + \frac{1}{\sqrt{2}}|--\rangle$$

- (a) At this time,  $S_{1z}$  is measured. What is the probability of finding  $-\hbar/2$ ? What is the state vector after this measurement?
  - (b) After the measurement in (a),  $S_{1x}$  is measured. What results can be found, and with what probabilities? Suppose that instead of finding  $-\hbar/2$  in (a), the measurement yielded  $+\hbar/2$ . Discuss the new result.
  - (c) Instead of either (a) or (b), we measure  $S_{1z}$  and  $S_{2z}$  simultaneously. What is the probability of finding the same result? What is the probability of finding opposite results?
2. Suppose that we define two multiparticle spin operators as  $S_a^{(1)} = S_{1z}$ , and  $S_b^{(2)} = \cos\theta S_{2z} + \sin\theta S_{2x}$ . Evaluate the expectation  $\langle S_a^{(1)} S_b^{(2)} \rangle$  for the singlet state,  $|\psi\rangle = \frac{1}{\sqrt{2}}[|+-\rangle - |-+\rangle]$ , and show that it is equal to  $-\frac{\hbar^2}{4} \cos\theta$ . Use only Dirac notation and algebra for this question.
  3. Explicitly calculate the result from the last question using the vector form of the singlet state, and the matrix form of the operator  $S_{1a} S_{2b}$ . Note that you will have to determine the form of each matrix first (hint: tensor product!).
  4. In class, we saw that a system of two spin-1/2 particles behaves like a system of total angular momentum  $J = 1$ , with  $M = -1, 0, +1$ , where  $M = m_1 + m_2$ . This system is doubly-degenerate for  $M = 0$ .
    - (a) Starting from the fact that the  $M = 1$  state is  $|++\rangle$ , derive the form of the states  $|J, M\rangle = |1, 0\rangle$  and  $|1, -1\rangle$  using the lowering operator  $J_- = S_{1-} + S_{2-}$ . Verify that you get the triplet we saw in class.
    - (b) Assume that the singlet state  $|J, M\rangle = |0, 0\rangle$  is a linear combination of the states  $|+-\rangle$  and  $|-+\rangle$ ,

$$|0, 0\rangle = \alpha|+-\rangle + \beta|-+\rangle$$

Using normalization of  $|0, 0\rangle$  and orthogonality with  $|1, 0\rangle$ , show that  $\alpha$  and  $\beta$  must give the proper singlet combination.