**Assignment #3**
Physics 322
Due: Wednesday 25 February 2009

Please answer all questions with complete solutions. All questions are of equal value.

1. As you are now aware, the Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ ,$$

are related to the operators that correspond to the spin components in the $x, y$, and $z$ directions. These matrices are elements of a matrix group called SU(2). This is the space of all $2 \times 2$ matrices that have unit determinant (the 'S' in SU(2)). and are also *unitary* (the 'U' in SU(2)). The SU(2) group plays an important role in two-state systems such as spin, and also in particle physics.

(a) Verify that these matrices are unitary.

(b) Verify that these matrices have unit determinant, *i.e.*$|\det[\sigma_i]| = 1$.

(c) Show that *any* matrix of the form $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ can be written as a linear combination of the matrices $\{I, \sigma_x, \sigma_y, \sigma_z\}$, where $I$ is the identity matrix. Hence, these 4 matrices can be understood to be a *basis* for all $2 \times 2$ matrices [Hint: show that the each coefficients $m_{ij}$ can be written as the linear combination of two of the four basis matrices].

2. Book problem 4.33.

3. A spin-1/2 particle is prepared in the general state

$$|+\rangle_u = \cos\frac{\theta}{2} e^{-i\frac{\phi}{2}} |+\rangle + \sin\frac{\theta}{2} e^{i\frac{\phi}{2}} |-\rangle$$

Show that the expectation values of the spin operators will be

$$\langle S_x \rangle = \frac{\hbar}{2} \sin\theta \cos\phi \ \ , \ \ \langle S_y \rangle = \frac{\hbar}{2} \sin\theta \sin\phi \ \ , \ \ \langle S_z \rangle = \frac{\hbar}{2} \cos\theta$$

The result implies that for large numbers of particles (which is what the average means!), the quantum mechanical spins become the components of a classical angular momentum vector with norm $\hbar/2$.

4. A system of two coupled spin-1/2 particles at time $t = 0$ is in the state

$$|\psi(0)\rangle = \frac{1}{2}|++\rangle + \frac{1}{2}|+-\rangle + \frac{1}{\sqrt{2}}|--\rangle$$

(a) At this time, $S_{1z}$ is measured. What is the probability of finding $-\hbar/2$? What is the state vector after this measurement?

(b) After the measurement in (a), $S_{1x}$ is measured. What results can be found, and with what probabilities? Suppose that instead of finding $-\hbar/2$ in (a), the measurement yielded $+\hbar/2$. Discuss the new result.

(c) Instead of either (a) or (b), we measure $S_{1z}$ and $S_{2z}$ simultaneously. What is the probability of finding the same result? What is the probability of finding opposite results?