restart

1.(a) First, we define the wavefunction for a particle in a box:

$$\text{psi} := (n, x, a) \rightarrow \text{sqrt}\left(\frac{2}{a}\right) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{a}\right)$$

$$(n, x, a) \rightarrow \sqrt{\frac{2}{a}} \sin\left(\frac{n \pi x}{a}\right) \quad \textbf{(1)}$$

Next, define the superposition:

$$pib := (x, y) \rightarrow \frac{1}{\text{sqrt}(2)} \cdot \text{psi}(1, x, 1) \cdot \text{psi}(2, y, 1) + \text{sqrt}\left(\frac{3}{10}\right) \cdot \text{psi}(2, x, 1) \cdot \text{psi}(3, y, 1) + \text{sqrt}\left(\frac{1}{5}\right) \cdot \text{psi}(3, x, 1) \cdot \text{psi}(1, y, 1)$$

$$(x, y) \rightarrow \frac{\psi(1, x, 1)\, \psi(2, y, 1)}{\sqrt{2}} + \sqrt{\frac{3}{10}}\, \psi(2, x, 1)\, \psi(3, y, 1) + \sqrt{\frac{1}{5}}\, \psi(3, x, 1)\, \psi(1, y, 1) \quad \textbf{(2)}$$

(b) The function is normalized:

$$\sqrt{2} \sin(\pi x) \sin(2 \pi y) + \frac{1}{5} \sqrt{30} \sin(2 \pi x) \sin(3 \pi y) + \frac{2}{5} \sqrt{5} \sin(3 \pi x) \sin(\pi y) \quad \textbf{(3)}$$

$$int\left(int\left(pib(x, y)^2, x = 0..1\right), y = 0..1\right)$$

(c) We can define the energy states:

$$1 \quad \textbf{(4)}$$

$$E := (nx, ny) \rightarrow \frac{\left(\left(\frac{nx}{1}\right)^2 + \left(\frac{ny}{1}\right)^2\right) \cdot \pi^2 \cdot hbar^2}{2 \cdot m}$$

$$(nx, ny) \rightarrow \frac{1}{2} \frac{\left(nx^2 + ny^2\right) \pi^2 hbar^2}{m} \quad \textbf{(5)}$$

So, the values for these states are:

$$E(1, 2); E(2, 3); E(3, 1)$$

$$\frac{5}{2} \frac{\pi^2 hbar^2}{m}$$

$$\frac{13}{2} \frac{\pi^2 hbar^2}{m}$$

$$\frac{5 \pi^2 hbar^2}{m} \quad \textbf{(6)}$$

Since they are all unique (no degeneracies), then we know that a measurement of this system will yield a distinct state.

2 (a) We introduce time dependence in the usual fashion:

$$pibt := (x, y, t) \rightarrow \frac{1}{\text{sqrt}(2)} \cdot \text{psi}(1, x, 1) \cdot \text{psi}(2, y, 1) \cdot \exp\left(-\frac{I \cdot E(1,1) \cdot t}{hbar}\right) + \text{sqrt}\left(\frac{3}{10}\right) \cdot \text{psi}(2, x, 1) \cdot \text{psi}(3, y, 1) \cdot \exp\left(-\frac{I \cdot E(2,3) \cdot t}{hbar}\right) + \text{sqrt}\left(\frac{1}{5}\right) \cdot \text{psi}(3, x, 1) \cdot \text{psi}(1, y, 1) \cdot \exp\left(-\frac{I \cdot E(3,1) \cdot t}{hbar}\right)$$

$$(x, y, t) \rightarrow \frac{\psi(1, x, 1)\, \psi(2, y, 1)\, e^{-\frac{I E(1, 1)\, t}{hbar}}}{\sqrt{2}} + \sqrt{\frac{3}{10}}\, \psi(2, x, 1)\, \psi(3, y, 1)\, e^{-\frac{I E(2, 3)\, t}{hbar}} + \sqrt{\frac{1}{5}}\, \psi(3, x, 1)\, \psi(1, y, 1)\, e^{-\frac{I E(3, 1)\, t}{hbar}}$$

$$(x, y, z, t) \rightarrow \frac{1}{\sqrt{2}} \left(\psi(1, x, 1)\, \psi(1, y, 2)\, \psi(1, z, 1)\, e^{\frac{I E(1, 2, 1)\, t}{hbar}} + \psi(2, x, 1)\, \psi(3, y, 2)\, \psi(2, z, 1)\, e^{\frac{I E(2, 3, 2)\, t}{hbar}}\right) \tag{8}$$

$$pibtst := (x, y, t) \rightarrow \frac{1}{\text{sqrt}(2)} \cdot \text{psi}(1, x, 1) \cdot \text{psi}(2, y, 1) \cdot \exp\left(+\frac{I \cdot E(1,1) \cdot t}{hbar}\right) + \text{sqrt}\left(\frac{3}{10}\right) \cdot \text{psi}(2, x, 1) \cdot \text{psi}(3, y, 1) \cdot \exp\left(+\frac{I \cdot E(2,3) \cdot t}{hbar}\right) + \text{sqrt}\left(\frac{1}{5}\right) \cdot \text{psi}(3, x, 1) \cdot \text{psi}(1, y, 1) \cdot \exp\left(+\frac{I \cdot E(3,1) \cdot t}{hbar}\right)$$

$$(x, y, t) \rightarrow \frac{\psi(1, x, 1)\, \psi(2, y, 1)\, e^{\frac{I E(1, 1)\, t}{hbar}}}{\sqrt{2}} + \sqrt{\frac{3}{10}}\, \psi(2, x, 1)\, \psi(3, y, 1)\, e^{\frac{I E(2, 3)\, t}{hbar}} + \sqrt{\frac{1}{5}}\, \psi(3, x, 1)\, \psi(1, y, 1)\, e^{\frac{I E(3, 1)\, t}{hbar}}$$

$$(x, y, z, t) \rightarrow pib(x, y, z, t)\, pibst(x, y, z, t) \tag{10}$$

(b) The probability density is

$$\text{rho} := (x, y, t) \rightarrow pibt(x, y, t) \cdot pibtst(x, y, t)$$

$$(x, y, t) \rightarrow pibt(x, y, t)\, pibtst(x, y, t) \tag{11}$$

$$\text{rho}(x, y, t)$$

$$\left(\sqrt{2}\, \sin(\pi x)\, \sin(2 \pi y)\, e^{-\frac{I \pi^2\, hbar\, t}{m}} + \frac{1}{5} \sqrt{30}\, \sin(2 \pi x)\, \sin(3 \pi y)\, e^{-\frac{\frac{13}{2} I \pi^2\, hbar\, t}{m}}\right. \tag{12}$$

$$+\frac{2}{5}\sqrt{5}\,\sin(3\,\pi\,x)\,\sin(\pi\,y)\,e^{-\frac{5\,I\,\pi^2\,hbar\,t}{m}}\Bigg)\Bigg(\sqrt{2}\,\sin(\pi\,x)\,\sin(2\,\pi\,y)\,e^{\frac{I\,\pi^2\,hbar\,t}{m}}$$

$$+\frac{1}{5}\sqrt{30}\,\sin(2\,\pi\,x)\,\sin(3\,\pi\,y)\,e^{\frac{\frac{13}{2}\,I\,\pi^2\,hbar\,t}{m}}+\frac{2}{5}\sqrt{5}\,\sin(3\,\pi\,x)\,\sin(\pi\,y)\,e^{\frac{5\,I\,\pi^2\,hbar\,t}{m}}\Bigg)$$

At time t=1, the probability is

$$int\left(int\left(int(rho(x,y,1),x=0.25..0.75),y=\frac{1}{2}..1\right),x=0..\frac{1}{2}\right)$$

$$0.2022043330-0.3417660965\cos\left(\frac{9.869604401\,hbar}{m}\right)^2 \qquad \textbf{(13)}$$

$$+0.3417660965\cos\left(\frac{9.869604401\,hbar}{m}\right)^4$$

This depends on hbar and the mass, but we can obtain numerical answers if we set them both to 1.

(c) Let's study the evolution of the system with the animate tool:

with(*plots*)

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*] **(14)**

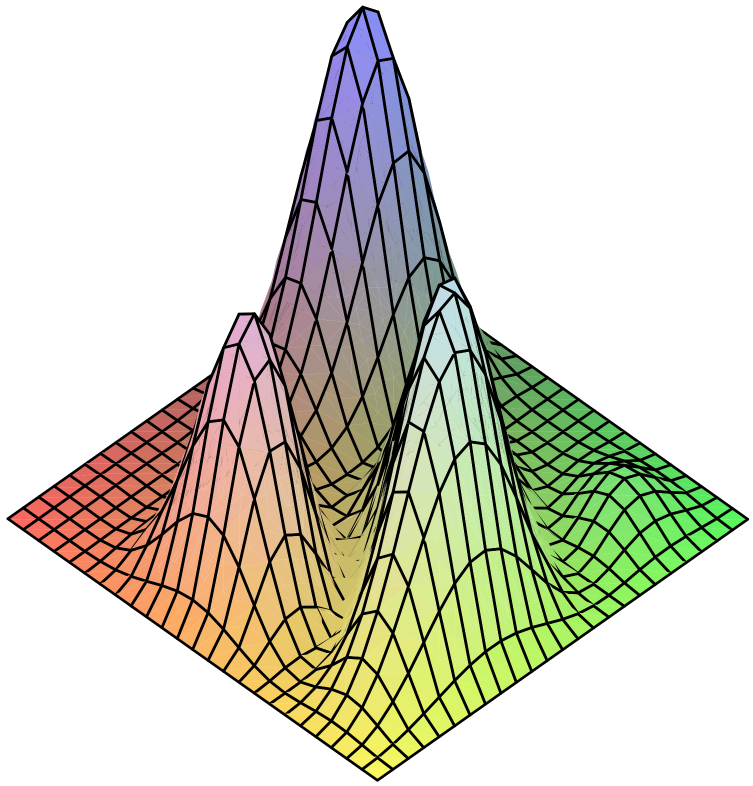
$m := 1;\ hbar := 1;$

$$1$$

$$1 \qquad \textbf{(15)}$$

$animate(\ plot3d,\ [\mathrm{rho}(x,y,t),\ x=0..1,\ y=0..1],\ t=0..2,\ frames=100\ );$

t = 0.



Note that the probability at the center never really increases, but there is considerable oscillatory behavior in the x- and y- directions.