Assignment #7, Physics 321
Due Date: Wednesday 09 December 2009

1. A particle is in a 2-D box of side-length $a = 1$ (*i.e.* it's a square box). Assume the particle is in a superposition of the states $(n_x, n_y) = (1,2), (2,3), (3,1)$ with respective probabilities $0.5, 0.3, 0.2$.

   (a) Write the wavefunction $\Psi(x, y)$ for the superposition. Be sure to define all of your terms.

   (b) Verify that it is normalized.

   (c) Will a measurement of the energy yield distinct states? (*i..e* is the system degenerate in any eigenstate or not?)

2. The system from the previous question is allowed to evolve in time.

   (a) Re-write the wavefunction $\Psi(x, y, t)$ to include the time-dependence. Be sure to define all terms used.

   (b) Determine the probability that, at $t = 1$, the particle is in the region $0 \leq x \leq a/2$, $a/2 \leq y \leq 1$.

   (c) Plot the probability density for $t = 0, 1, 2$. You can use the animate function again to watch it move around!

3. In class we discussed theories with extra spatial dimensions in which these dimensions are compactified, *i.e.* curled up into a tiny d-dimensional ball of radius R. A free particle in the extra dimensions has as its wavefunctions

$$\phi_n(y) = a_n \sin\left(\frac{ny}{R}\right) + b_n \cos\left(\frac{ny}{R}\right), \quad \phi_n(y) = \phi_n(y + 2\pi R)$$

   where the latter condition exemplifies the fact that the extra dimensions are periodic. The wavefunction for a particle in a 2-D box, where one is compactified, is thus

$$\Psi_{m,n}(x, y) = \psi_m(x)\phi_n(y)$$

   with $\psi_m(x)$ being the usual 1-D particle-in-a-box wavefunction.

   (a) Show that the normalization condition on $\Psi_{m,n}(x, y)$ implies $a_n^2 + b_n^2 = \frac{1}{\pi R}$. Assume the box spans $0 \leq x \leq 1$ in the non-compactified dimension.

   (b) We derived the extra-dimensional energy eigenvalues for this particle to be $E_n = \frac{n^2\hbar^2}{2mR^2}$, but this assumes the particle to have mass. In actual fact, the only particles that are believed to exist in the extra dimensions are gravitons (the carriers of the gravitational force), which are massless. Derive an expression for the particle's momentum p_n and energy E_n by determining the wavelength λ_n of the wavefunction $\phi_n(y)$.