Assignment #4

Physics 321

Due: Wednesday 14 October 2009

Please include print-outs of your Maple/Mathematica results, and as always be sure you discuss your solutions (otherwise no points will be given for the question).

1. The free particle wavefunction solution to Schrödinger's Equation is

$$\psi(x) = A\sin\left(\frac{px}{\hbar}\right) + B\cos\left(\frac{px}{\hbar}\right)$$

   (a) Show that for a suitable choice of complex coefficients c_1, c_2, the function

$$\psi(x) = c_1 e^{\frac{ipx}{\hbar}} + c_2 e^{-\frac{ipx}{\hbar}}$$

   is also a solution.

   (b) Determine how c_1 and c_2 relate to A and B.

   (c) Using the Schrödinger equation, show that the values of the energy and momentum are related in the usual classical sense.

2. You are told that the function $\psi(x) = Ax^2$ is supposed to represent a particle confined to the region $0 \leq x \leq 1$, where A is the appropriate normalization constant (that you must determine). Determine whether or not this can really be a wavefunction, in the sense that it respects the Heisenberg Uncertainty Principle.

3. A particle in a box (*aka* a particle in an infinite well) of width $a = 1$ is in the superposition state

$$\psi(x) = \sqrt{\frac{1}{3}}\phi_1(x) + \sqrt{\frac{2}{3}}\phi_2(x)$$

   where $\phi_1(x)$ and $\phi_2(x)$ are the usual solutions to the Schrödinger equation for this case,

$$\phi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

   (a) Verify that $\psi(x)$, as it is written, is normalized.

   (b) What is the probability that a measurement will yield a particle of energy $E = \frac{2\pi^2\hbar^2}{m}$?

   (c) Plot the probability distribution, and use it to estimate the average position of the particle.

   (d) Double-check your guess in part (c) by explicitly calculating the expectation value of the position operator, $\langle X \rangle$.

4. Show that the wavefunction in question 3 satisfies the Heisenberg Uncertainty Principle, $(\Delta p)(\Delta x) \geq \frac{\hbar}{2}$.