Assignment #2
Physics 321
Due: Monday 21 September 2009

Please answer all questions with complete solutions. All questions are of equal value.

1. Suppose a quantum mechanical system is described by the wavefunction

$$|\psi\rangle = 0.2|\psi_1\rangle + 0.4|\psi_2\rangle + 0.8|\psi_3\rangle \tag{1}$$

where the states $|\psi_i\rangle$ are orthogonal (*i.e.* form an orthonormal basis).

(a) Explain what is wrong with this wavefunction.

(b) Correct the expression if the problem exists with the $|\psi_1\rangle$ component. What is the associated probability of measuring this state?

(c) How do the associated probabilities of measuring each state change if we multiply each eigenstate $|\psi_j\rangle$ by a different complex phase $e^{i\alpha_j}$, *i.e.*

$$|\psi\rangle = 0.2|\psi_1\rangle e^{i\alpha_1} + 0.4|\psi_2\rangle e^{i\alpha_2} + 0.8|\psi_3\rangle e^{i\alpha_3}$$

2. For the following combinations of state vectors, indicate which is a number (complex or real), another state vector, an operator, or none of the above. Be sure to explain why it is what you say it is (otherwise you'll lose a lot of points!). If it's a number, don't worry about the actual value.

   (a) $\langle\alpha|\beta\rangle$
   (b) $|\alpha\rangle\langle\alpha|\beta\rangle$
   (c) $|\alpha\rangle\langle\beta|$
   (d) $|\langle\alpha|\beta\rangle|^2$
   (e) $\langle\alpha|A|\alpha\rangle$, where A is an operator

3. Let $z = 1 + e^{i\theta}$ be a complex number.

   (a) Convince yourself that $|z|^2 = z^*z \neq z^2$.
   (b) Show that $|z|^2 = 2 + 2\cos\theta$.

4. In class, we discussed the different representations for polarization states of the photon, either linear $\{|x\rangle, |y\rangle\}$, or circular $\{|L\rangle, |R\rangle\}$. For linear polarization, we can write the state as

$$|\theta\rangle = \cos\theta|x\rangle \pm \sin\theta|y\rangle$$

while for circular we have

$$|\theta\rangle = \frac{1}{\sqrt{2}}\left[e^{i\theta}|L\rangle + e^{-i\theta}|R\rangle\right]$$

(a) Show that these two expressions are equivalent under the transformation

$$|L\rangle = \frac{1}{\sqrt{2}}\left[|x\rangle + i|y\rangle\right] \quad , \quad |R\rangle = \frac{1}{\sqrt{2}}\left[|x\rangle - i|y\rangle\right]$$

(b) Based on the orthogonality conditions $\langle x|x\rangle$, $\langle x|y\rangle$, and $\langle y|y\rangle$, deduce the corresponding orthogonality relationships between $|L\rangle$ and $|R\rangle$.