Assignment #1

Physics 321

Due: Wednesday 09 Sep 2009

Please answer all questions with complete solutions. All questions are of equal value.

For some of these questions, you'll need to refresh your memory on double-slit interference and single slit diffraction. We'll talk about it in class, but you should also revisit your introductory texts, too. Alternatively, you can go to the *Hyperphysics* website (linked in on the website).

1. There are three fundamental constants in the universe: the speed of light c, Planck's constant $\hbar$, and Newton's constant G. In the SI, the value and units of these constants are:

$$c = 2.99 \times 10^8 \; \frac{\text{m}}{\text{s}} \quad ; \quad \hbar = 1.05 \times 10^{-34} \; \frac{\text{kg} \cdot \text{m}^2}{\text{s}} \quad ; \quad G = 6.67 \times 10^{-11} \; \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$$

   Find a product/combination of the constants (each raised to a particular power) that has dimensions of ...:

   (a) ... *mass*. This number is called the *Planck mass*, m_{P}.
   (b) ... *time*. This number is called the *Planck time*, t_{P}.
   (c) ... *length*. This number is called the *Planck length*, l_{P}.

2. (a) In E&M, you learned that the electrostatic force between two charges is an inverse-square law, $F_e = \frac{kq_pq_e}{r^2}$. Suppose instead that the attractive force between the electron and the proton in the hydrogen atom are given by some power law $F_e = kq_pq_er^\beta$, where $\beta \neq 0$ can be any number (this might happen in theories where the photon can move into extra spatial dimensions!). In class, we derived the quantization conditions for the hydrogen energy levels in the case $\beta = -2$. Repeat the procedure for general β, and show that the energy level for the n^{th} orbit is

$$E_n = \left(\frac{\hbar^2 n^2}{m}\right)^{\frac{\beta+1}{\beta+3}} k^{\frac{2}{\beta+3}} \left(\frac{1}{2} + \frac{1}{\beta+1}\right)$$

   (b) Verify that the usual quantization conditions are recovered for $\beta = -2$ [Hint: you'll first need to find the potential energy that gives the force law in question.]

3. In optics, a diffraction pattern is formed when light of wavelength λ reflects off a grating made up of small slits of width a. The wave-like nature of the electron was experimentally observed in the Davisson-Germer experiment, which

involved the scattering of (relatively) low-energy electrons off a crystal lattice (basically a diffraction grating for electrons). A detector positioned away from the crystal showed interspersed regions of strong electron impacts, as well as low.

(a) Suppose the atoms in the lattice are spaced at roughly 2.15 $\AA$, and the strongest peak of the electron distribution is at an angle of $50°$ to the incident beam. Based on your answer in (a), determine the associated wavelength of the particle.

(b) The electron beam is produced by ionizing a metal and accelerating the ejected particles through a potential difference ΔV. If the electrons are non-relativistic, what would be their corresponding deBroglie wavelength λ_{dB} if $\Delta V = 54\ V$? You'll need to review some E&M for this, as well as diffraction!

4. Quantum effects such as interference are readily observed in mesoscopic and subatomic particles, but not in macroscopic masses. This is due to the fact that the fundamental scaling of quantum mechanics – Planck's Constant – is extremely small ($h = 6.63 \times 10^{-34}$ J·s).

   (a) Suppose Planck's Constant was actually much larger, say $h = 10^{-6}$ J·s. A trail of ants marches toward two holes in a wall, with the intent of getting to a tasty treat on the other side. Determine where you could place the food so that the ants *wouldn't* walk over it. Assume each ant has a mass of 0.1 g, and walks at a speed of 0.5 m/s. The two holes are separated by a distance of 10 cm.

   (b) How big would Planck's Constant have to be in order for a single-file line of people walking through a doorway (50 cm wide) to end up in groups 1 m apart on the wall opposite the door? Assume the width of the room (door to wall) is 5 m, the people walk at 1.5 m/s, and they each way about 60 kg. [Hint: the angular spacing of successive minima in single-slit diffraction is given by $\sin\theta \approx m\lambda/D$, where m is an integer and D is the slit width.]