Assignment #10 solutions

1. **Problem 10.14** The hiker starts at an altitude of \(y_i = -85.0\) m and ends at an altitude of \(y_f = 4420\) m, so the total change in elevation is \(\Delta y = y_f - y_i = 4420 - (-85.0) = 4505\) m. The change in gravitational potential energy is thus

\[
\Delta U = mg\Delta y = (65.0)(9.8)(4505) = 2.87 \times 10^6\ J.
\]

2. **Problem 10.19** The total vertical displacement of the car is \(\Delta y = -50\) m (since it rolls down the hill), so the total change in potential energy is \(\Delta U = mg\Delta y\). By conservation of energy, this is also equal to the (negative) total change in kinetic energy, \(\Delta K = -\Delta U\) (what’s lost in potential energy goes into kinetic energy!). Since the car starts from rest \((v_i = 0\) m/s), the total change in kinetic energy is \(\Delta K = \frac{1}{2}mv_f^2\), so we can find the velocity at the bottom of the hill:

\[
\frac{1}{2}mv_f^2 = -mg\Delta y \implies v_f = \sqrt{-2g\Delta y} = \sqrt{-2(9.8)(-50)} = 31.3\ m/s
\]

Note that the mass doesn’t play a part!

3. **Problem 10.53** The sledder starts on a hill that is \(y_i = 30\) m above the ground, and wants to get to the top of the other hill \((y_f = 42\) m). If he starts from rest \((v_i = 0\) m/s), then the highest point he will reach on the other hill is \(30\) m. Thus, he will need extra energy to make it an additional \(\Delta y = y_f - y_i = 12\) m – in fact, he will need an amount \(\Delta U = mg\Delta y\). This energy will come from an initial kinetic energy, \(K_i = \frac{1}{2}mv_i^2\). If he just reaches the top of the hill, then \(K_f = 0\) J, and so \(\Delta K = K_f - K_i = -\frac{1}{2}mv_i^2\). So, by conservation of energy:

\[
\Delta K = -\Delta U \implies v_i\sqrt{2g\Delta y} = \sqrt{(2)(9.8)(12)} = 15.3\ m/s
\]

Note that this answer is not only independent of \(m\), but also independent of \(g\)!

4. **Problem 10.59** This problem is exactly the same as the stunt car portion of yesterday’s lab! By conservation of energy, the person will acquire an amount of kinetic energy \(\Delta K = \Delta U = -mg\Delta y_1\), where \(\Delta y_1 = -3.0\) m (negative because he slide drops 3 m). Their velocity exiting the slide will be \(v = \sqrt{2g\Delta y}\), which as usual is independent of the mass. After this point, it’s all kinematics! What happens in \(\hat{x}\) stays in \(\hat{x}\), what happens in \(\hat{y}\) stays in \(\hat{y}\). The two are tied together by time \(\Delta t\) only. We are told this velocity is completely horizontal, so the person will fall into the water a horizontal distance \(\Delta x = v\Delta t\). Here, \(\Delta t\) is the time require for the person to fall a vertical distance \(\Delta y_2 = -1.2\) m. Using the kinematic equation \(\Delta y_2 = \frac{1}{2}g\Delta t^2\), we find \(\Delta t = \sqrt{\frac{2\Delta y_2}{g}}\), so coupled with the horizontal equation we get

\[
\Delta x = v\Delta t = \sqrt{2g\Delta y_1}\sqrt{\frac{2\Delta y_2}{g}} = \sqrt{4\Delta y_1\Delta y_2} = 2\sqrt{(3.0)(-1.2)} = 3.8\ m.
\]

Note that this answer is not only independent of \(m\), but also independent of \(g\)!