1. (a) If the collision is elastic, then we know that both kinetic energy and momentum are conserved. We saw in class that if this is true, then the following relationship between initial and final velocities holds:

\[ v_{1i} + v_{1f} = v_{2i} + v_{2f} \]

So, if \( v_{1i} = +30 \text{ m/s} \) and \( v_{1f} = -30 \text{ m/s} \), then

\[ v_{2i} + v_{2f} = 0 \]

and so we must have \( v_{2i} = -v_{2f} \). That is, the initial and final velocities of the racket are equal but opposite – which means that the speed of the racket doesn’t change.

(b) By conservation of momentum, we know that

\[
\begin{align*}
\vec{p}_{1i} + \vec{p}_{2i} &= \vec{p}_{1f} + \vec{p}_{2f} \\
m_1v_{1i} + m_2v_{2i} &= -m_1v_{1f} + m_2v_{2f} \\
m_1v_{1i} - m_1v_{1f} &= -m_1v_{2i} + m_2v_{2f} \\
(0.05)(30 - (-30)) &= -m_1v_{2i} + m_2v_{2f} \\
1.5 &= -m_1v_{2i} + m_2v_{2f}
\end{align*}
\]

Since we know that \( v_{2i} = -v_{2f} \), we can substitute this in and solve for \( v_{2i} \):

\[
\begin{align*}
1.5 &= -2m_1v_{2i} \\
v_{2i} &= \frac{-1.5}{-2(0.5)} = 1.5 \text{ m/s}
\end{align*}
\]

2. If the object is dropped from a height of 2 m, this means that initially it has \( U_i = mgh = (1.0)(9.8)(2.0) = 19.6 \text{ J} \) of energy available. So, by the time it reaches the floor, all of this energy has been converted into kinetic energy. When the object hits the floor and stops, \boxed{all of this energy} has been lost. That is, the floor took 19.6 J of kinetic energy from the mass (because the floor stopped the mass from moving – it did work).

3. Since the initial momentum of particle 1 is \( \vec{p}_{1i} = (2\hat{x}) \text{ kg m/s} \), and the initial momentum of particle 2 is \( \vec{p}_{2i} = (4\hat{y}) \text{ kg m/s} \), the total initial momentum must be \( \vec{p}_i = \vec{p}_{1i} + \vec{p}_{2i} = (2\hat{x} + 4\hat{y}) \text{ kg m/s} \). By conservation of momentum, this must be the total final momentum, \( \vec{p}_f = \vec{p}_i \). This means that the total final momentum must have components \( p_{fx} = 2 \text{ kg m/s} \), and \( p_{fy} = 4 \text{ kg m/s} \). These components constrain the final momenta of each particle, since the sum of their momentum components in each direction must add to those \( p_{fx} \) and \( p_{fy} \):

\[ p_{fx} = p_{1fx} + p_{2fx} \]
\[ p_{fy} = p_{1fy} + p_{2fy} \]

Since the final momentum of particle 1 is 3 kg m/s at 45°, it has components \( p_{1fx} = 3 \cos(45) = 2.1 \text{ kg m/s} \), and \( p_{1fy} = 3 \sin(45) = 2.1 \text{ kg m/s} \). This means we can find the momentum components for particle 2:

\[ p_{2fx} = p_{fx} - p_{2fx} = 2 - 2.1 = -0.1 \text{ kg m/s} \]
\[ p_{2fy} = p_{fy} - p_{2fy} = 4 - 2.1 = 1.9 \text{ kg m/s} \]

We can also express this as a magnitude+angle:

\[ p_{2f} = \sqrt{(0.1)^2 + (1.9)^2} = 1.9 \text{ kg m/s} \]

with

\[ \theta = \arctan\left(\frac{0.1}{1.9}\right) = 3° \] to the left of the vertical

4. A child \((m = 30 \text{ kg})\) runs at a speed of 3 m/s toward a tire swing of mass 10 kg. The child jumps on the swing.

(a) Since the child holds on to the swing, this is an \textit{inelastic collision}.

(b) Since kinetic energy is lost in an inelastic collision, the velocity of the child+tire is less than the incoming velocity of the child. By conservation of momentum,

\[ p_f = p_i \]
\[ (m_c + m_t)v_f = m_cv_c + m_tv_t \]
\[ v_f = \frac{m_cv_c}{m_c + m_t} \]
\[ v_f = \frac{(30)(3)}{30 + 10} \]
\[ v_f = 2.3 \text{ m/s} \]

(c) The swing will rise to a height where all the kinetic energy is converted into potential energy (conservation of energy!). So:

\[ U_f = K_i \]
\[ (m_c + m_t)gh = \frac{1}{2}(m_c + m_t)v^2 \]
\[ h = \frac{v^2}{2g} \]
\[ h = \frac{2.3^2}{(2)(9.8)} \]
\[ h = 0.26 \text{ m} \] (1)

Thus, the swing will rise by \(26 \text{ cm}\).