Assignment #4 solutions  
Physics 253 (Mureika)

1. If his take-off velocity is $\vec{v}_i$ at an angle $\theta$, then the x- and y-components are $v_{xi} = v_i \cos \theta$ and $v_{yi} = v_i \sin \theta$. We know that the total vertical displacement is $\Delta x = 8.95 \text{ m}$, so $\Delta x = v_{xi}t$, where $t$ is the total time he is in the air. Since projectile motion is symmetric about the highest point of the trajectory, then we can say the time required to go up ($t_{up}$) is equal to half of the entire time. We can calculate $t_{up}$ using the equation

\[
v_{yf} - v_{yi} = gt_{up}
\]

\[
0 - v_{yi} = gt_{up}
\]

\[
t_{up} = \frac{-v_{yi}}{g}
\]

So, $t = 2t_{up}$, and we can re-insert this into the equation for the x-displacement:

\[
\Delta x = v_{xi} - \frac{2v_{yi}}{g}
\]

\[
\Delta x = \frac{-2v_{xi}v_{yi}}{g}
\]

\[
\Delta x = \frac{-2v_i^2 \sin \theta \cos \theta}{g}
\]

where we have substituted the component equations from before. Rearranging this to solve for $v_i$ (the magnitude of the take-off velocity), we find

\[
v_i = \sqrt{-\frac{g\Delta x}{2 \sin \theta \cos \theta}} = \sqrt{-\frac{(-9.8)(8.95)}{2 \sin(20) \cos(20)}}
\]

\[
v_i = 11.68 \text{ m/s}
\]

So, his take-off velocity vector was $\vec{v} = 11.68 \text{ m/s at } 20^\circ$, or in component form

\[
\vec{v} = [10.98 \hat{x} + 4.00 \hat{y}] \text{ m/s}
\]

(b) The maximum height he rose off the ground is

\[
\Delta y = \frac{v_{yi}^2}{2g} = \frac{0-(3.99)^2}{2(-9.8)} = 0.81 \text{ m}
\]
The child running west will have a displacement \( d_1 = \frac{1}{2}a_1t^2 = (0.5)(0.75)(4)^2 = 6.0 \text{ m} \). The child running north-east will have a total displacement of \( d_2 = \frac{1}{2}a_2t^2 = (0.5)(1.25)(4)^2 = 10.0 \text{ m} \). We wish to find the length of the line \( d_t \), which is the distance between them. We note that the total displacement between them in the \( x \)-direction is simply \( d_1 + d_{2x} \), where \( d_{2x} \) is the \( x \)-component of the displacement of child 2. The total \( y \)-displacement between them is \( d_{2y} \). Since \( d_{2x} = d_2 \cos(45) = 7.1 \text{ m} \), and \( d_{2y} = d_2 \cos(45) = 7.1 \text{ m} \), the total components of their displacement are \( d_{tx} = d_1 + d_{2x} = 13.1 \text{ m} \), and \( d_{ty} = d_{2y} = 7.1 \text{ m} \). So, the total displacement is
\[
|d_t| = \sqrt{d_{tx}^2 + d_{ty}^2} = \sqrt{(13.1)^2 + (7.1)^2} = 14.9 \text{ m}.
\]
Alternatively, we could use the vector form of the kinematic equations. Child 2 is accelerating both in the \( x \)- and \( y \)-directions, with acceleration components \( a_{2x} = a_2 \cos(45) = 0.88 \text{ m/s}^2 \), and \( a_{2y} = a_2 \sin(45) = 0.88 \text{ m/s}^2 \). So, the child’s displacement vector components are \( d_{2x} = \frac{1}{2}a_{2x}t^2 = 7.1 \text{ m} \), and \( d_{2y} = \frac{1}{2}a_{2y}t^2 = 7.1 \text{ m} \).

3. The ball has a total horizontal displacement of 15 ft, or \( \Delta x = 15 \text{ ft} \left( \frac{1 \text{ m}}{3.333 \text{ ft}} \right) = 4.5 \text{ m} \). Similarly, the total vertical displacement is from \( y_i = 2.0 \text{ m} \) to \( y_f = 10 \text{ ft} = 3.0 \text{ m} \), or \( \Delta y = 1.0 \text{ m} \). When the ball reaches the hoop, its vertical velocity component must be \( v_{yf} = 0 \). Thus, we know that the initial \( y \)-component of the velocity is \( v_{yi} = \sqrt{-2g\Delta y} = \sqrt{-2(-9.8)(1.0)} = 4.4 \text{ m/s} \). The ball is in the air for \( t = \frac{-v_i}{g} = \frac{-4.4}{-9.8} = 0.45 \text{ s} \), so the \( x \)-component of the velocity must be \( v_x = \frac{\Delta x}{t} = \frac{4.5}{0.45} = 10.0 \text{ m/s} \).

Thus, the initial velocity vector is \( \vec{v}_i = [10.0\hat{x} + 4.5\hat{y}] \text{ m/s} \).

4. Since \([v] = \frac{m}{s} \) and \([r] = m \), the quantity \( A \) must have units \([A] = \frac{[v^2]}{[v]} = \frac{m^2/s^2}{m/s} = \frac{m}{s^2} \).

It is an acceleration (in fact, it’s the centripetal acceleration for an object undergoing uniform circular motion. We’ll see this again soon!)