Assignment #1 solutions

1. Chapter 2, Problem 21
   (a) The pike accelerates from rest at a rate of \( a = \frac{\Delta v}{\Delta t} = \frac{4.0 \text{ m/s}}{0.11 \text{ s}} = 36.4 \text{ m/s}^2 \).

   (b) The distance the pike travels in this time is given by the kinematic equation \( \Delta x = v_i t + \frac{1}{2} a t^2 \), where \( v_i = 0, a = 36.4 \text{ m/s}^2 \), and \( t = 0.11 \text{ s} \). Thus, \( \Delta x = \frac{1}{2} (36.4)(0.11^2) = 0.22 \text{ m} \).

2. Chapter 2, Problem 27
   (a) Let’s call \( x_0 = 0 \text{ m}, t_0 = 0 \text{ s} \) the point at which the driver sees the deer. Since the brakes are not applied for another \( t_1 = 0.5 \text{ s} \), we know the car is still moving at \( v_1 = 20 \text{ m/s} \) when it starts slowing down. In this time, it has traveled an additional distance \( \Delta x_1 = v_1 t_1 = (20)(0.5) = 10 \text{ m} \). The deer is now only 25 m away, so the car must slow to a stop in no more than this distance for the deer to be safe: \( \Delta x_2 = x_2 - x_1 = 25 \text{ m} \). Since we know the car’s maximum acceleration is \( a_1 = -10 \text{ m/s}^2 \), we can use our third kinematic equation to solve for the displacement:

\[
\frac{v_i^2}{2} - \frac{v_1^2}{2} = 2a_1 \Delta x_2 \quad \implies \quad \Delta x_2 = \frac{v_i^2 - v_1^2}{2a_1} = \frac{0 - 20^2}{-20} = 20 \text{ m}
\]

Horray! From the time the driver sees the deer, the car travels a total distance of \( \Delta x_1 + \Delta x_2 = 10 + 20 = 30 \text{ m} \). The deer is saved!

   (b) To determine how fast the driver could have been traveling without killing the poor deer, we need to find the velocity \( v_i \) such that \( \Delta x = \Delta x_1 + \Delta x_2 = 35 \text{ m} \) exactly. We can combine the two values into one equation, noting that the reaction time \( 0.5 \text{ s} = \frac{1}{2} \text{ s} \):

\[
\Delta x_1 = v_i t_1 = \frac{1}{2} v_i \quad ; \quad \Delta x_2 = \frac{0 - v_i^2}{2a_1}
\]

\( \implies \Delta x = \frac{v_i}{2} - \frac{v_i^2}{2a} = \frac{v_1 a - v_i^2}{2a} = 35 \text{ m} \)

Plugging in the value of the acceleration, we get \( 35 = \frac{-10v_i - v_i^2}{-20} \) which can be rearranged to give \( 0 = v_i^2 + 10v_i - 700 \). This is a quadratic equation in \( v_i \), which has solutions \( v_i = \frac{-10 \pm \sqrt{10^2 - 4(-700)}}{2} = -5 \pm 5\sqrt{29} \). The maximum initial velocity must be \( v_i = -5 + 5\sqrt{29} = 21.9 \text{ m/s} \).

3. Chapter 2, Problem 29
   The model of the 100 m race can be divided into two phases: (1) the acceleration
phase, and (2) the coasting phase. We know that the sprinter starts from rest \( v_0 = 0 \text{ m/s} \) at \( t_0 = 0 \text{ s}, x_0 = 0 \text{ m} \), and accelerates for a time \( t_1 = 2.14 \text{ s} \) to a maximum velocity of \( v_1 = 11.2 \text{ m/s} \) at this point. This means that his acceleration is \( a = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0} = \frac{11.2 - 0}{2.14 - 0} = 5.23 \text{ m/s}^2 \). The acceleration phase thus takes place over a distance \( \Delta x_1 = v_0 t_1 + \frac{1}{2} a t_1^2 = (0)(2.14) + \frac{1}{2}(5.23)(2.14)^2 = 11.98 \text{ meters} \). The coasting phase is therefore \( \Delta x_2 = 100 - x_1 = 88.02 \text{ m} \) long, so if the sprinter travels at a constant speed of \( v = 11.2 \text{ m/s} \) for this distance, it will take \( t_2 = \frac{\Delta x_2}{v} = \frac{88.02}{11.2} = 7.86 \text{ s} \). The total race time is therefore \( t = t_1 + t_2 = 2.14 + 7.86 = 10.00 \text{ s} \). Not quite a match for Usain Bolt!

In reality, a sprinter actually accelerates through to about 60 m and reaches a top velocity of around 12 m/s. He doesn’t hold his top speed for the last 40 m, but rather slows down to around 11.3-10.8 m/s by 100 m.

4. Chapter 2, Problem 30
(a) The ball-bearing must be in free fall for \( t = 4 \text{ s} \), which means that it can’t hit the ground before this time. If it starts from rest, we know that

\[
\Delta y = \frac{1}{2} g t^2 = \frac{1}{2}(-9.8)(4)^2 = -78.4 \text{ m}
\]

Note that the vertical displacement is negative, which means that the ball is dropping!

(b) The impact velocity can be calculated easily using the fact that \( g = \frac{v_f - v_i}{t} \), which gives \( v_f = g t = (-9.8)(4.0) = -39.2 \text{ m/s} \). Again, the velocity is negative because it is moving down! Note that this is a whopping \( 39.2 \text{ m/s} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) \left( \frac{1 \text{ mile}}{1600 \text{ m}} \right) = 88 \text{ mph} \)!