

APPENDIX A: PROOFS

PROOF OF PROPOSITION 1 [It can also be shown that Condition (i) or (ii) alone is sufficient for this proposition to hold]:

The Lagrangian function for this problem is

$$(A1) \quad L = v(y) - f(y - \phi, \alpha) - c(\phi - \eta, \beta) + \lambda_y (\bar{y} - y) + \lambda_\phi (\bar{y} - \phi).$$

From (A1) one derives the following first order Kuhn-Tucker conditions

$$(A2) \quad \frac{\partial L}{\partial y} = v_1(y) - f_1(y - \phi, \alpha) - \lambda_y \leq 0$$

$$(A3) \quad \frac{\partial L}{\partial y} = 0$$

$$(A4) \quad \frac{\partial L}{\partial \phi} = f_1(y - \phi, \alpha) - c_1(\phi - \eta, \beta) - \lambda_\phi \leq 0$$

$$(A5) \quad \frac{\partial L}{\partial \phi} = 0$$

$$(A6) \quad \frac{\partial L}{\partial \lambda_y} = \bar{y} - y \geq 0$$

$$(A7) \quad \frac{\partial L}{\partial \lambda_y} = 0$$

$$(A8) \quad \frac{\partial L}{\partial \lambda_\phi} = \bar{y} - \phi \geq 0$$

$$(A9) \quad \frac{\partial L}{\partial \lambda_\phi} = 0.$$

Part A:

Consider the following two cases.

Case 1:  $y^* < \bar{y}$ . Equations (A2), (A6) and (A7) imply  $v_1(y^*) - f_1(y^* - \phi^*, \alpha) \leq 0$ . Since  $v_1(y^*) > 0$ , this implies  $f_1(y^* - \phi^*, \alpha) > 0$  and  $\phi^* < y^*$ . Equations (A4), (A8) and (A9) imply  $f_1(y^* - \phi^*, \alpha) - c_1(\phi^* - \eta, \beta) \leq 0$  which implies  $c_1(\phi^* - \eta, \beta) > 0$  and  $\eta < \phi^*$ . Thus,  $\eta < \phi^* < y^* < \bar{y}$ .

Case 2:  $y^* = \bar{y}$ . Proving  $\phi^* < \bar{y}$  by contradiction, suppose  $\phi^* = \bar{y}$ . Then (A4) and (A5) imply  $f_1(y^* - \phi^*, \alpha) - c_1(\phi^* - \eta, \beta) = \lambda_\phi \geq 0$ . Note, however, that  $f_1(y^* - \phi^*, \alpha) = 0$ , implying  $c_1(\phi^* - \eta, \beta) \leq 0$ . But  $c_1(\phi^* - \eta, \beta) > 0$  since  $\phi^* = \bar{y} > \eta$  which establishes a contradiction. Thus,

$\phi^* < y^*$ . If this is the case,  $\lambda_\phi = 0$  by (A8) and (A9), and (A4) becomes  $f_1(y^* - \phi^*, \alpha) - c_1(\phi^* - \eta, \beta) \leq 0$ . Note that  $f_1(y^* - \phi^*, \alpha) > 0$ , so  $c_1(\phi^* - \eta, \beta) < 0$  and  $\eta < \phi^*$ . Thus,  $\eta < \phi^* < y^* = \bar{y}$ . For this corner solution, note by (A2), (A3), (A6) and (A7) that  $v_1(y^*) - f_1(y^* - \phi^*, \alpha) = \lambda_y \geq 0$  which, with (A4), (A5), (A8) and (A9), implies  $v_1(y^*) \geq f_1(y^* - \phi^*, \alpha) = c_1(\phi^* - \eta, \beta)$ .

Part B:

When  $\alpha = 1$  and  $y \neq \phi$ ,  $f(\cdot) = \infty$  and  $u(\cdot) = -\infty$ . Therefore, the dictator sets  $y = \phi$  and  $f(y^* - \phi^*, 1) = 0$ . The Lagrangian simplifies to  $L = v(y) - c(y - \eta, \beta) + \lambda_y(\bar{y} - y)$  and the first order conditions become

$$(A2') \quad \frac{\partial L}{\partial y} = v_1(y) - c_1(y - \eta, \beta) - \lambda_y \leq 0$$

and equations (A3), (A6) and (A7). If  $y^* = 0$ , (A6) and (A7) imply  $\lambda_y = 0$  and (A2') becomes  $v_1(0) - c_1(-\eta, \beta) \leq 0$ . But  $v_1(0) > 0$  and  $c_1(-\eta, \beta) < 0$ , so  $y^* \neq 0$ . If  $y^* \in (0, \bar{y})$ , (A2'), (A3), (A6) and (A7) imply  $v_1(y^*) - c_1(y^* - \eta, \beta) = 0$  and  $y^* > \eta$ . If  $y^* = \bar{y}$ , then  $\phi^* = \bar{y} > \eta$ . Thus,  $\eta < \phi^* = y^* \leq \bar{y}$ .

Part C:

When  $\beta = 1$  and  $\phi \neq \eta$ ,  $c(\cdot) = \infty$  and  $u(\cdot) = -\infty$ . Thus, the dictator chooses  $\phi = \eta$  and  $c(\phi^* - \eta, 1) = 0$ . The Lagrangian becomes  $L = v(y) - f(y - \eta, \alpha) + \lambda_y(\bar{y} - y)$  and the first order conditions are reduced to (A2), (A3), (A6) and (A7). If  $y^* = 0$ , (A6) and (A7) imply  $\lambda_y = 0$  and (A2) becomes  $v_1(0) - f_1(-\eta, \alpha) \leq 0$ , a contradiction. Hence,  $y^* \neq 0$ . If  $y^* \in (0, \bar{y})$ , (A2), (A3), (A6) and (A7) imply  $v_1(y^*) - f_1(y^* - \eta, \alpha) = 0$  and  $y^* > \eta$ . If  $y^* = \bar{y}$ ,  $\phi^* = \eta < y^*$ . Thus,  $\eta = \phi^* < y^* \leq \bar{y}$ .

Part D:

As demonstrated for parts B and C above, when  $\alpha = 1$ , the dictator sets  $y = \phi$ , and when  $\beta = 1$ , the dictator sets  $\phi = \eta$ . Thus,  $\eta = \phi^* = y^* < \bar{y}$ .

Parts A through D prove the general claim of Proposition 1.

PROOF OF PROPOSITION 2 [Condition (i) or (ii) alone is sufficient for this to hold]:

Note that boundary solutions, that is, cases in which  $\partial L / \partial \lambda_y = \lambda_y = 0$  or  $\partial L / \partial \lambda_\phi = \lambda_\phi = 0$ , are subsumed under other cases below, the particular one depending on the direction of change in  $y^*$  or  $\phi^*$  precipitated by the change in  $\eta$ ,  $\alpha$  or  $\beta$ . Where the Kuhn-Tucker conditions apply, then, we consider only cases in which  $\partial L / \partial \lambda_y > 0$  or  $\lambda_y > 0$  and  $\partial L / \partial \lambda_\phi > 0$  or  $\lambda_\phi > 0$  below.

Part A:

Case 1:  $\alpha, \beta \in (0,1)$ ;  $y^* < \bar{y}$ . By Proposition 1A,  $\eta < \phi^* < y^*$ , and from (A2) through (A9),

$$(A10) \quad \frac{\partial u}{\partial y} = v_1(y^*) - f_1(y^* - \phi^*, \alpha) = 0$$

$$(A11) \quad \frac{\partial u}{\partial \phi} = f_1(y^* - \phi^*, \alpha) - c_1(\phi^* - \eta, \beta) = 0.$$

The sufficient second order conditions imply, by the implicit function theorem, that (A10) and (A11) can be solved for the choice functions  $y^*(\eta, \alpha, \beta)$  and  $\phi^*(\eta, \alpha, \beta)$ . Substituting these into (A10) and (A11) yields

$$(A12) \quad v_1(y^*(\eta, \alpha, \beta)) - f_1(y^*(\eta, \alpha, \beta) - \phi^*(\eta, \alpha, \beta), \alpha) = 0$$

$$(A13) \quad f_1(y^*(\eta, \alpha, \beta) - \phi^*(\eta, \alpha, \beta), \alpha) - c_1(\phi^*(\eta, \alpha, \beta) - \eta, \beta) = 0.$$

Differentiating (A12) and (A13) with respect to  $\eta$  results in

$$\begin{aligned} v_{11} \frac{\partial y^*}{\partial \eta} - f_{11} \frac{\partial y^*}{\partial \eta} + f_{11} \frac{\partial \phi^*}{\partial \eta} &= 0 \\ f_{11} \frac{\partial y^*}{\partial \eta} - f_{11} \frac{\partial \phi^*}{\partial \eta} - c_{11} \frac{\partial \phi^*}{\partial \eta} + c_{11} &= 0. \end{aligned}$$

In matrix form this is

$$\begin{bmatrix} v_{11} - f_{11} & f_{11} \\ f_{11} & -f_{11} - c_{11} \end{bmatrix} \begin{bmatrix} \partial y^* / \partial \eta \\ \partial \phi^* / \partial \eta \end{bmatrix} = \begin{bmatrix} 0 \\ -c_{11} \end{bmatrix}.$$

The Hessian determinant is

$$|H| = (v_{11} - f_{11})(-f_{11} - c_{11}) - (f_{11})^2 = -v_{11} f_{11} - v_{11} c_{11} + f_{11} c_{11} > 0.$$

Applying Cramer's Rule,

$$\frac{\partial y^*}{\partial \eta} = \frac{\begin{vmatrix} 0 & f_{11} \\ -c_{11} & -f_{11} - c_{11} \end{vmatrix}}{|H|} = \frac{c_{11} f_{11}}{|H|} > 0$$

$$\frac{\partial \phi^*}{\partial \eta} = \frac{\begin{vmatrix} v_{11} - f_{11} & 0 \\ f_{11} & -c_{11} \end{vmatrix}}{|H|} = \frac{c_{11}(f_{11} - v_{11})}{|H|} > 0.$$

Case 2:  $\alpha, \beta \in (0,1)$ ;  $y^* = \bar{y}$ .  $\frac{\partial y^*}{\partial \eta} = 0$  since  $y^* = \bar{y}$ . From Proposition 1A,  $\eta < \phi^* < \bar{y}$ , and

from (A5), (A8), (A9), equation (A4) becomes

$$(A14) \quad f_1(\bar{y} - \phi^*(\eta, \alpha, \beta), \alpha) - c_1(\phi^*(\eta, \alpha, \beta) - \eta, \beta) = 0,$$

substituting  $\phi^*(\eta, \alpha, \beta)$  for  $\phi^*$ . Differentiating (A14) with respect to  $\eta$  yields

$$-f_{11} \frac{\partial \phi^*}{\partial \eta} - c_{11} \frac{\partial \phi^*}{\partial \eta} + c_{11} = 0, \text{ or } \frac{\partial \phi^*}{\partial \eta} = \frac{c_{11}}{f_{11} + c_{11}} > 0.$$

Case 3:  $\alpha = 1, \beta \in (0,1)$ ;  $y^* < \bar{y}$ . From Proposition 1B,  $\eta < \phi^* = y^*$ , and it follows that  $f(y^* - \phi^*, 1) = 0$ . The objective function becomes  $u(y, \eta, \beta) = v(y) - c(y - \eta, \beta)$ , and the first order condition, substituting  $y^*(\eta, \beta)$  for  $y^*$ , is

$$(A15) \quad v_1(y^*(\eta, \beta)) - c_1(y^*(\eta, \beta) - \eta, \beta) = 0.$$

Differentiating (A15) with respect to  $\eta$ ,  $v_{11} \frac{\partial y^*}{\partial \eta} - c_{11} \frac{\partial y^*}{\partial \eta} + c_{11} = 0$ , or  $\frac{\partial y^*}{\partial \eta} = \frac{c_{11}}{c_{11} - v_{11}} > 0$ . Also,

$$\frac{\partial \phi^*}{\partial \eta} = \frac{\partial y^*}{\partial \eta} > 0 \text{ since } \phi^* = y^*.$$

Case 4:  $\alpha = 1, \beta \in (0,1)$ ;  $y^* = \bar{y}$ . By Proposition 1B,  $\eta < \phi^* = y^* = \bar{y}$ . Hence,  $\frac{\partial y^*}{\partial \eta} = \frac{\partial \phi^*}{\partial \eta} = 0$ .

Case 5:  $\alpha \in (0,1), \beta = 1$ ;  $y^* < \bar{y}$ . From Proposition 1C,  $\eta = \phi^* < y^*$ , thus  $c(\phi^* - \eta, 1) = 0$ .

From (A3), (A6), and (A7), we write equation (A2)

$$(A16) \quad v_1(y^*(\eta, \alpha)) - f_1(y^*(\eta, \alpha) - \eta, \alpha) = 0,$$

substituting  $y^*(\eta, \alpha)$  for  $y^*$ . Differentiating (A16) with respect to  $\eta$ ,  $v_{11} \frac{\partial y^*}{\partial \eta} - f_{11} \frac{\partial y^*}{\partial \eta} + f_{11} = 0$ .

Therefore,  $\frac{\partial y^*}{\partial \eta} = \frac{f_{11}}{f_{11} - v_{11}} > 0$ , and  $\frac{\partial \phi^*}{\partial \eta} = 1 > 0$  since  $\phi^* = \eta$ .

Case 6:  $\alpha \in (0,1), \beta = 1$ ;  $y^* = \bar{y}$ . From Proposition 1C,  $\eta = \phi^* < y^* = \bar{y}$ . It then follows that  $\frac{\partial y^*}{\partial \eta} = 0$  and  $\frac{\partial \phi^*}{\partial \eta} = 1 > 0$ .

Case 7:  $\alpha = \beta = 1$ . By Proposition 1D,  $\eta = \phi^* = y^* < \bar{y}$ . Thus,  $\frac{\partial y^*}{\partial \eta} = \frac{\partial \phi^*}{\partial \eta} = 1 > 0$ .

Part B:

Case 1:  $\alpha \in (0,1]$ ,  $\beta \in (0,1)$ ;  $y^* < \bar{y}$ . Note that when  $\alpha = 1$  it is only meaningful to consider  $d\alpha < 0$ . Then the new  $\alpha \in (0,1)$  and, by Proposition 1A,  $\eta < \phi^* < y^*$ . Differentiating (A12) and (A13) with respect to  $\alpha$  yields  $v_{11} \frac{\partial y^*}{\partial \alpha} - f_{11} \frac{\partial y^*}{\partial \alpha} + f_{11} \frac{\partial \phi^*}{\partial \alpha} - f_{12} = 0$ , and  $f_{11} \frac{\partial y^*}{\partial \alpha} - f_{11} \frac{\partial \phi^*}{\partial \alpha} + f_{12} - c_{11} \frac{\partial \phi^*}{\partial \alpha} = 0$ . In matrix form this is

$$\begin{bmatrix} v_{11} - f_{11} & f_{11} \\ f_{11} & -f_{11} - c_{11} \end{bmatrix} \begin{bmatrix} \partial y^* / \partial \alpha \\ \partial \phi^* / \partial \alpha \end{bmatrix} = \begin{bmatrix} f_{12} \\ -f_{12} \end{bmatrix}.$$

As before,  $|H| > 0$ . Hence,  $\frac{\partial y^*}{\partial \alpha} = \frac{-f_{12}c_{11}}{|H|} < 0$  and  $\frac{\partial \phi^*}{\partial \alpha} = \frac{-v_{11}f_{12}}{|H|} > 0$ .

Case 2:  $\alpha \in (0,1]$ ,  $\beta \in (0,1)$ ;  $y^* = \bar{y}$ . As above  $d\alpha < 0$  if  $\alpha = 1$ .  $\frac{\partial y^*}{\partial \alpha} = 0$  since  $y^* = \bar{y}$ .

Differentiating (A14) with respect to  $\alpha$  yields  $-f_{11} \frac{\partial \phi^*}{\partial \alpha} + f_{12} - c_{11} \frac{\partial \phi^*}{\partial \alpha} = 0$ , or  $\frac{\partial \phi^*}{\partial \alpha} = \frac{f_{12}}{f_{11} + c_{11}} > 0$ .

Case 3:  $\alpha \in (0,1)$ ,  $\beta = 1$ ;  $y^* < \bar{y}$ . From Proposition 1C,  $\eta = \phi^* < y^*$ . Differentiating (A16) with respect to  $\alpha$  yields  $v_{11} \frac{\partial y^*}{\partial \alpha} - f_{11} \frac{\partial y^*}{\partial \alpha} - f_{12} = 0$ , or  $\frac{\partial y^*}{\partial \alpha} = \frac{f_{12}}{v_{11} - f_{11}} < 0$ .  $\frac{\partial \phi^*}{\partial \alpha} = 0$  since  $\phi^* = \eta$ .

Case 4:  $\alpha \in (0,1)$ ,  $\beta = 1$ ;  $y^* = \bar{y}$ . By Proposition 1C  $\eta = \phi^* < y^* = \bar{y}$ , so  $\frac{\partial y^*}{\partial \alpha} = \frac{\partial \phi^*}{\partial \alpha} = 0$ .

Case 5:  $\alpha = \beta = 1$ . This is only meaningful when  $d\alpha < 0$ , and then it is captured by cases 3 and 4 above. Thus,  $\frac{\partial y^*}{\partial \alpha} < (=) 0$  as  $y^* < (=) \bar{y}$ , and  $\frac{\partial \phi^*}{\partial \alpha} = 0$ .

Part C:

Case 1:  $\alpha \in (0,1)$ ,  $\beta \in (0,1]$ ;  $y^* < \bar{y}$ . We consider  $d\beta < 0$  when  $\beta = 1$  in which case the new  $\beta \in (0,1)$  and by Proposition 1A,  $\eta < \phi^* < y^*$ . Differentiating (A12) and (A13) with respect to  $\beta$  gives  $v_{11} \frac{\partial y^*}{\partial \beta} - f_{11} \frac{\partial y^*}{\partial \beta} + f_{11} \frac{\partial \phi^*}{\partial \beta} = 0$  and  $f_{11} \frac{\partial y^*}{\partial \beta} - f_{11} \frac{\partial \phi^*}{\partial \beta} - c_{11} \frac{\partial \phi^*}{\partial \beta} - c_{12} = 0$ . As before,  $|H| > 0$ , and now  $\frac{\partial y^*}{\partial \beta} = \frac{-f_{11}c_{12}}{|H|} < 0$  and  $\frac{\partial \phi^*}{\partial \beta} = \frac{(v_{11} - f_{11})c_{12}}{|H|} < 0$ .

Case 2:  $\alpha \in (0,1)$ ,  $\beta \in (0,1]$ ;  $y^* = \bar{y}$ . As before, consider  $d\beta < 0$  if  $\beta = 1$ . By Proposition 1A,  $\eta < \phi^* < y^* = \bar{y}$ , and  $\frac{\partial y^*}{\partial \beta} = 0$  since  $y^* = \bar{y}$ . Differentiating (A14) with respect to  $\beta$  gives  $-f_{11} \frac{\partial \phi^*}{\partial \beta} - c_{11} \frac{\partial \phi^*}{\partial \beta} - c_{12} = 0$ , or  $\frac{\partial \phi^*}{\partial \beta} = \frac{-c_{12}}{f_{11} + c_{11}} < 0$ .

Case 3:  $\alpha = 1$ ,  $\beta \in (0,1)$ ;  $y^* < \bar{y}$ . By Proposition 1B,  $\eta < \phi^* = y^*$ . Differentiating (A15) with respect to  $\beta$  yields  $v_{11} \frac{\partial y^*}{\partial \beta} - c_{11} \frac{\partial y^*}{\partial \beta} - c_{12} = 0$ , or  $\frac{\partial y^*}{\partial \beta} = \frac{c_{12}}{v_{11} - c_{11}} < 0$ .  $\frac{\partial \phi^*}{\partial \beta} = \frac{\partial y^*}{\partial \beta} < 0$  since  $\phi^* = y^*$ .

Case 4:  $\alpha = 1$ ,  $\beta \in (0,1)$ ;  $y^* = \bar{y}$ . By Proposition 1B,  $\eta < \phi^* = y^* = \bar{y}$ , so  $\frac{\partial y^*}{\partial \beta} = \frac{\partial \phi^*}{\partial \beta} = 0$ .

Case 5:  $\alpha = \beta = 1$ . This is meaningful only when  $d\beta < 0$  in which case it is described by the cases 3 and 4 above. Therefore,  $\frac{\partial y^*}{\partial \beta} = \frac{\partial \phi^*}{\partial \beta} < (=) 0$  as  $y^* = \phi^* < (=) \bar{y}$ .

PROOF OF PROPOSITION 3 [Conditions (i) and (ii) are sufficient]:

The Lagrangian function for this problem is

$$(A16) \quad L = -f(y - \phi, \alpha) - c(\phi - \eta, \beta) + \lambda_y (\bar{y} - y) + \lambda_\phi (\bar{y} - \phi).$$

The first order conditions are equations (A3), (A4), (A5), (A6), (A7), (A8), (A9) and

$$(A2'') \quad \frac{\partial L}{\partial y} = -f_1(y - \phi, \alpha) - \lambda_y \leq 0.$$

The second order conditions follow from the concavity of  $u_b(\cdot)$ .

Case 1:  $\alpha, \beta \in (0,1)$ . If  $y^* = 0$ , then by (A2''), (A3), (A6) and (A7),  $-f_1(-\phi^*, \alpha) \leq 0$  which implies  $\phi^* = 0$  and  $f_1(y^* - \phi^*, \alpha) = 0$ . This fact and equations (A4), (A5), (A8) and (A9) imply  $-c_1(-\eta, \beta) \leq 0$  which implies  $\eta = 0$ . Hence,  $\eta = \phi^* = y^*$ . If  $y^* \in (0, \bar{y})$ , then by (A2''), (A3), (A6) and (A7),  $-f_1(y^* - \phi^*, \alpha) = 0$  which implies  $y^* = \phi^*$ . By (A4), (A5), (A8) and (A9),  $-c_1(\phi^* - \eta, \beta) = 0$  which implies  $\phi^* = \eta$ . Thus,  $\eta = \phi^* = y^*$ . If  $y^* = \bar{y}$ , by (A2''), (A3), (A6) and (A7),  $-f_1(y^* - \phi^*, \alpha) = \lambda_y \geq 0$  which implies  $y^* \leq \phi^*$ . But then  $\phi^* = \bar{y}$  and  $f_1(y^* - \phi^*, \alpha) = 0$ . By (A4), (A5), (A8) and (A9),  $-c_1(\phi^* - \eta, \beta) = \lambda_\phi \geq 0$  which implies  $\phi^* \leq \eta$  and  $\eta = \bar{y}$ . This establishes a contradiction or, if we drop condition (iii), proof more generally that  $\eta = \phi^* = y^*$  even when  $\eta = \bar{y}$ .

Case 2:  $\alpha = 1$ ,  $\beta \in (0, 1)$ . Then set  $y = \phi$  and  $f(y^* - \phi^*, 1) = 0$ . The Lagrangian becomes  $L = -c(\phi - \eta, \beta) + \lambda_\phi (\bar{y} - \phi)$  and the first order conditions become (A5), (A8), (A9) and

$$(A4') \quad -c_1(\phi - \eta, \beta) - \lambda_\phi \leq 0.$$

If  $\phi^* = 0$ , then these equations imply  $-c_1(-\eta, \beta) \leq 0$  which implies  $\eta = 0$ . Thus,  $\phi^* = \eta$ . If  $\phi^* \in (0, \bar{y})$ , then (A4'), (A5), (A8) and (A9) imply  $-c_1(\phi^* - \eta, \beta) = 0$  which implies  $\phi^* = \eta$ . If  $\phi^* = \bar{y}$ , then we can conclude that  $-c_1(\phi^* - \eta, \beta) = \lambda_\phi \geq 0$  which implies  $\phi^* \leq \eta$  and  $\eta = \bar{y}$ , a contradiction or, dropping condition (iii), the more general proof.

Case 3:  $\alpha \in (0, 1)$ ,  $\beta = 1$ . Then set  $\phi = \eta$  and  $c(\phi^* - \eta, 1) = 0$ . The Lagrangian becomes  $L = -f(y - \eta, \alpha) + \lambda_y (\bar{y} - y)$  and the first order conditions are (A2''), (A3), (A6) and (A7). If  $y^* = 0$ , it follows that  $-f_1(-\eta, \alpha) \leq 0$  which implies  $\eta = 0$  and  $y^* = \eta$ . If  $y \in (0, \bar{y})$ , then  $-f_1(y^* - \eta, \alpha) = 0$  and  $y^* = \eta$ . If  $y^* = \bar{y}$  then  $-f_1(y^* - \eta, \alpha) = \lambda_y \geq 0$  which implies  $y^* \leq \eta$  and  $\eta = \bar{y}$ , again a contradiction or, dropping Condition (iii), a more general proof.

Case 4:  $\alpha = \beta = 1$ . Then set  $\eta = \phi = y$  and  $f(y^* - \phi^*, 1) = c(\phi^* - \eta, 1) = 0$ .

PROOF OF PROPOSITION 4 [Condition (i) alone suffices for this proposition]:

The Lagrangian function for this problem is

$$(A17) \quad L = -f(y - \phi^*, \alpha) - c(\phi^* - \eta, \beta) + \lambda_y (\bar{y} - y).$$

The first order conditions are equations (A3), (A6), (A7) and

$$(A2''') \quad \frac{\partial L}{\partial y} = -f_1(y - \phi^*, \alpha) - \lambda_y \leq 0.$$

The second order conditions follow from the concavity of  $u_d(\cdot)$ .

Case 1:  $\alpha \in (0, 1)$ . If  $y^{**} = 0$ , then from the first order conditions  $-f_1(-\phi^*, \alpha) \leq 0$  which implies  $\phi^* = 0$ . If  $y^{**} \in (0, 1)$ , then  $-f_1(y^{**} - \phi^*, \alpha) = 0$  which implies  $y^{**} = \phi^*$ . If  $y^{**} = \bar{y}$ , then  $-f_1(y^{**} - \phi^*, \alpha) = \lambda_y \geq 0$  which implies  $y^{**} \leq \phi^*$  and  $\phi^* = \bar{y}$ . Thus,  $y^{**} = \phi^*$ .

Case 2:  $\alpha = 1$ . Then set  $y = \phi^*$  since  $f(y - \phi^*, 1) = \infty$  when  $y \neq \phi^*$ .

## APPENDIX B: INSTRUCTIONS

This Appendix contains the different versions of instructions which were read to the subjects at the beginning of each phase of the experiment. Differences in wording for Rooms A and B are indicated with parentheses or brackets, respectively, i.e., (A)/[B], and differences for the discretionary and exogenous treatments are indicated with chevrons and braces, respectively, i.e., <discretionary>/{exogenous}. Except for the version number in Roman numerals, the instructions are as they appeared on handouts to the students. The version used for a specific room, phase and treatment are summarized below followed by the instructions.

### Version of Instructions

<u>Room</u>	<u>Phase</u>	<u>Discretionary Differences</u>		<u>Exogenous Differences</u>	
		<u>Standard</u>	<u>Benevolent</u>	<u>Standard</u>	<u>Benevolent</u>
A	1	I	I	I	I
	2	II	III	II	III
	Further 2			IV	
B	1	I	I	I	I
	2	II	III	II	III
C	1		V	I*	V
	2			III*	
D	1			I*	
	2			III*	

\*For simplicity, Rooms C and D were told that they had been labeled A and B, respectively, and were read those rooms' instructions from the Benevolent Dictator treatment.

### I Phase 1 Instructions Room (A)[B]

You have been asked to participate in an experiment. For your participation today we have paid you \$3 in cash. You may earn an additional amount of money, which will also be paid to you in cash at the end of the experiment.

In this experiment each of you will be paired with a different person who is in another room. We will call this room Room (A)[B] and the other room Room (B)[A]. That person has also been paid \$3 to participate and is being read these same instructions which are being read to you. You will not be told who those people are either during or after the experiment, and they will not be told who you are either during or after the experiment. You and your counterpart in Room (B)[A] make up what we will call a "pair." You will notice that there are other people in the same room with you who are also participating in the experiment. You will not be matched with any of these people. The decisions that they make will have absolutely no effect on you nor will any of your decisions affect them.

There are two stages to this experiment which will be conducted as follows. In the first phase you and your counterpart in Room (B)[A] are asked to perform separately a task which involves preparing letters for mailing. For each envelope you correctly produce, a certain money credit is earned. Similarly, for each envelope your counterpart in Room (B)[A] produces, a certain credit is earned. You will be informed of the exact amount of these credits after the first phase is over, but they will not be less than 25 cents per letter and across all subjects will average 50 cents per letter. The earnings from each person in this room and the person's counterpart in the other room will be credited to an account which is assigned to that pair. Each person in both rooms is working under the same conditions, with the same kind of materials and producing the same kind of letters.

After this task is complete, we come to the second phase in which the money is distributed. All of the money credited to each pair's account will be distributed in cash to that pair. A given member of a pair cannot be guaranteed any specific amount, however, since that will depend on the decision of an arbitrarily chosen person. The details of the manner of this distribution will be provided after the first phase is complete.

Now we will explain the details of the first phase. Each of you has in front of you a study carrel, a stack of <twenty>{ten} white letters, <twenty>{ten} blue letters and <twenty>{ten} envelopes and to the right of your seat a sealed box. The tasks you are to complete are now demonstrated by my assistant. You are to fold the letters in thirds and put one white and one blue letter separately in each envelope. Then you close the envelope but do not seal it and put it through one of the two slots at each end of the box. You may accomplish these tasks in any way you wish as long as the basic results are obtained. Everyone will be given <five>{seven} minutes to prepare the letters. The more letters a pair correctly prepares, the more money is credited to that pair's account. When I call "Time!" you must cease all activity: leave the remaining materials on your desk - any envelope not already in your box when time is called does not count. If you have any questions, please ask them now.

**II**  
**Phase 2 Instructions**  
**Room (A)[B]**

In a moment the results of the task will be made available to you. The exact same information will also be made available to your counterpart in Room (B)[A]. (The people here in Room A have been arbitrarily chosen to decide how the total will be distributed between you and your counterparts in Room B.) [Your counterparts in Room A have been arbitrarily chosen to decide how the total will be distributed between you and them.] The form you are about to receive will indicate separately for you and for your counterpart how many envelopes each of you correctly produced, the credit per envelope for each person, (and) how much total has been credited to your and your counterpart's account[, and how much, if any, of the total your counterpart has allocated to him- or herself and how much, if any, he or she has allocated to you]. {The credit per envelope may differ for you and for your counterpart. This difference is completely arbitrary: in other words, any difference in this credit does not reflect any difference in the quantity or quality of work by you or your counterpart or any difference in your working conditions.} (The form you receive will also include spaces for indicating how much, if any, of the total you wish to allocate to yourself and how much, if any, you wish to allocate to your counterpart in Room B. This decision is completely up to you and is confidential: only the experimenter will know who made this decision. Just make sure that the total you allocate to yourself and your counterpart equals the total available in the account. Place the form in the envelope provided. You will be given five minutes to make your decision. Then an assistant will come by to pick up the envelopes and will begin to prepare your payments.) Before leaving, each of you will be called individually to the front to receive your payment confidentially in a closed envelope after which you will be free to go immediately. (After all of your payments have been made and you are gone, we will prepare the payments for your counterparts in Room B and will provide them with the results of the task and with any payments which you have allocated to them.)

**III**  
**Phase 2 Instructions**  
**Room (A)[B]**

In a moment the results of the task will be made available to you. The exact same information will also be made available to your counterpart in Room (B)[A]. In addition, this same information has been made available to participants in this experiment who are seated in a

third room we will call Room C. Each individual in Room C has been matched with a pair from Rooms A and B. These people have not been and will not be told who you are, either during and after the experiment, and you will not be told who these people are, either during or after the experiment. The people in Room C have been arbitrarily chosen to decide how the total will be distributed between you and your counterparts in Room (B)[A]. The form you are about to receive will indicate separately for you and for your counterpart in Room (B)[A] how many envelopes each of you correctly produced, the credit per envelope for each person, how much total has been credited to your and your Room (B)[A] counterpart's account, and how much, if any, of the total your counterpart in Room C has allocated to you and how much, if any, he or she has allocated to your Room (B)[A] counterpart. {The credit per envelope may differ for you and for your counterpart. This difference is completely arbitrary: in other words, any difference in this credit does not reflect any difference in the quantity or quality of work by you or your counterpart or any difference in your working conditions.} Before leaving, each of you will be called individually to the front to receive your payment confidentially in a closed envelope and to sign a receipt after which you are free to go immediately.

**IV**  
**Further Phase 2 Instructions**  
**Room A**

While your payments are being prepared, you will be asked to make another decision about the distribution of money. Consider two other rooms, which we will call Rooms C and D, in which individuals are asked to prepare letters according to the same instructions and under the same conditions as you. As with you and your counterpart in Room B, each person in Room C is paired with someone in Room D. The people here in Room A have been arbitrarily chosen also to decide how the total will be distributed between the pairs in Rooms C and D. For this decision you will be paid an additional \$3 beyond your other payments. Each of the persons in this room has been matched with a pair in Rooms C and D. You are about to receive a form which will indicate separately for your counterparts in Room C and D the number of correctly produced envelopes, the credit per envelope for each person and how much total has been credited to C and D's account. The credit per envelope may differ for your counterpart in Room C and your counterpart in Room D. As before, this difference is completely arbitrary: it does not reflect any differences in quantity, quality or working conditions. The form you receive will also include spaces for indicating how much, if any, of the total you wish to allocate to your counterpart in Room C and how much, if any, you wish to allocate to your counterpart in Room D. This decision is completely up to you and is confidential: only the experimenter will know who made this decision. As before, just make sure that the total you allocate to both equals the total available in the account. Then you will place the form in an envelope which is provided. You will be given five minutes to make your decision. Then an assistant will come by to pick up the envelopes. Finally, you will receive your payments for the earlier task with your counterpart in Room B and for this decision. After all of your payments have been made and you are gone, we will prepare the payments for your counterparts in Rooms B, C and D, and will provide them with the results of their respective tasks and with any payments which you have allocated to them.

**V**  
**Instructions**  
**Room C**

You have been asked to participate in an experiment. For your participation today we have paid you \$3 in cash. You will receive an additional \$5 which will also be paid to you in cash at the end of the experiment.

In this experiment each of you is matched with two different people in two different rooms. We will call those rooms Room A and Room B and this room Room C. Those people have also been paid \$3 to participate. You will not be told who those people are either during or after the experiment, and they will not be told who you are either during or after the experiment. The people with whom you have been matched in Rooms A and B are your "pair." You will notice that there are other people in the same room with you who are also participating in the experiment. You will not be matched with any of these people. The decisions that they make will have absolutely no effect on you nor will any of your decisions affect them.

There are two stages to this experiment which will be conducted as follows. In the first phase your pair in Rooms A and B has been asked to perform separately a task which involves preparing letters for mailing. For each envelope an individual correctly produced, a certain money credit was earned. They were told that they would be informed of the exact amount of this credit after the first phase but that it would not be less than 25 cents per letter and across all subjects would average 50 cents per letter. The earnings from both individuals were credited to an account which is assigned to that pair. Each person in both rooms was working under the same conditions, with the same kind of materials and producing the same kind of letters.

After this task is complete, we come to the second phase in which the money is distributed. All of the money credited to each pair's account will be distributed in cash to that pair. The people in this room have been arbitrarily chosen to decide on the allocation of the money credited to their counterparts in Rooms A and B.

Now we will explain the details of the first phase which has been completed in the other rooms. Each person in Rooms A and B had in front of him- or herself a study carrel such as yours, a stack of <twenty>{ten} white letters, <twenty>{ten} blue letters and <twenty>{ten} envelopes and next to the seat a sealed box. The tasks they were assigned are now demonstrated by my assistant. They were to fold the letters in thirds and put one white and one blue letter separately in each envelope. Then they were to close the envelope but not seal it and put it through one of the two slots at each end of the box. They could accomplish these tasks in any way they wished as long as the basic results were obtained. Everyone was given <five>{seven} minutes to prepare as many letters as possible. The more letters a pair prepared, the more money was credited to that pair's account. When "Time!" was called all activity had to cease: any envelope not already in the box when time was called did not count.

In a moment the results of the task will be made available to you. The exact same information will also be made available to your counterparts in Rooms A and B. Now the people here in Room C are to decide how the total will be distributed among your counterparts in Rooms A and B. The form you are about to receive will indicate separately for your counterparts in Room A and in Room B how many envelopes each correctly produced, the credit per envelope for each person and how much total has been credited to their account. {The credit per envelope may differ for your counterparts in Rooms A and B. This difference is completely arbitrary: in other words, any difference in this credit does not reflect any difference in the quantity or quality of work by your counterpart in Room A versus that of your counterpart in Room B or any difference in their working conditions.} The form you receive will also include spaces for indicating how much, if any, of the total you wish to allocate to your counterpart in Room A and how much, if any, you wish to allocate to your counterpart in Room B. This decision is completely up to you and is confidential: only the experimenter will know who made this decision. Just make sure that the total you allocate to the two of them equals the total available in the account. Then you will place the form in an envelope which is provided. You will be given five minutes to make your decision. Then an assistant will come by to pick up the envelopes and will begin to prepare your payments. Before leaving, each of you will be called individually to the front to receive your payment after which you will be free to go immediately. After all of your payments have been made and you are gone, we will prepare the payments for your

counterparts in Rooms A and B and will provide them with the results of the task and with any payments which you have allocated to them. If you have any questions, please ask them now.

APPENDIX C: FORMS

This appendix contains the decision forms and questionnaires filled out by the subjects. Below are a summary of the versions used for specific rooms and treatments and a reproduction of the demographic questions posed on questionnaires. These are followed by copies of the decision forms and questionnaires provided to subjects. The only other changes here versus the original forms are the addition of the version number in Roman numerals, the deletion of the subject ID in the lower right of all forms and the specific differences in wording, where applicable, noted on the forms.

**Version of Forms**

<u>Room</u>	<u>Discretionary Differences</u>		<u>Exogenous Differences</u>	
	<u>Standard</u>	<u>Benevolent</u>	<u>Standard/Double</u>	<u>Benevolent</u>
A	I, II	III	I, II, IV, V	III
B	VI	III	VI	III
C		IV, VII	III*	IV, VII
D			III*	

\*In keeping with the instructions, Rooms C and D were called A and B, respectively, on these forms.

**Demographic Questions**

Forms II, III, VI and VII also requested at the bottom of the sheet the following demographic information:

Please circle or fill in the following information about yourself:

Freshman      Sophomore      Junior      Senior      Graduate

Caucasian      Latino/Hispanic      African-American      Asian      Other: \_\_\_\_\_

Male      Female

Major: \_\_\_\_\_

Age: \_\_\_\_\_

## I Results of Task

	<u>Number of envelopes correctly completed</u>	×	<u>Credit per envel.</u>	=	<u>Money credited to account</u>
You		×		=	\$
Your counterpart		×		=	\$
	_____				
Total					\$ <span style="border: 1px solid black; display: inline-block; width: 100px; height: 20px; vertical-align: middle;"></span>
					Box A

## Decision Form

Now you are to allocate the money in Box A. Please indicate the amount of this total you wish to allocate to:

Yourself	\$ _____
Your counterpart	\$ _____
Total	\$ <span style="border: 1px solid black; display: inline-block; width: 100px; height: 20px; vertical-align: middle;"></span>
	Box B

Make sure that the total in Box A is the same as the total in Box B. Please place this form in the envelope provided and wait silently for someone to come by and pick it up.

## II Room A

### Questionnaire

Why did you choose to allocate the money between yourself and your counterpart as you did?

### III

Note: Differences in language for Rooms A and B are indicated with bold parentheses and brackets, respectively, i.e., (A)/[B].

#### Room (A)[B]

#### Results

	Number of envelopes <u>correctly completed</u>	×	Credit <u>per envel.</u>	=	Money <u>credited to account</u>
(You (A))[Your counterpart (A)]		×		=	\$
(Your counterpart (B))[You (B)]		×		=	\$
	_____				_____
Total					\$

Of this total, your counterpart in Room C decided to allocate the following amounts to:

(You (A))[Your counterpart (A)]     \$ \_\_\_\_\_  
(Your counterpart (B))[You (B)]     \$ \_\_\_\_\_

#### Questionnaire

Consider the following situation: suppose you are in the position of your counterpart in Room C: you decide how the money is divided between Person A in Room A and Person B in Room B assuming the same conditions and the same results as in the task you just completed. Please indicate the amount you would allocate to:

Person A     \$  
Person B     \$  
Total         \$

#### IV

Note: Differences in wording for the Room A Double Dictator and the Room C Benevolent Dictator are indicated with parentheses and brackets, respectively, i.e., (Room A Double)/[Room B Benevolent] Dictator.

#### Results of Task for Rooms (C)[A] and (D)[B]

	Number of envelopes <u>correctly completed</u>	×	Credit <u>per envel.</u>	=	Money <u>credited to account</u>
Room (C)[A] counterpart		×		=	\$
Room (D)[B] counterpart		×		=	\$
_____					
Total					<div style="border: 1px solid black; padding: 5px; display: inline-block;">\$</div>
					Box A

#### Decision Form

Now you are to allocate the money in Box A. Please indicate the amount of this total you wish to allocate to:

Room (C)[A] counterpart		\$ _____
Room (D)[B] counterpart		\$ _____
Total		<div style="border: 1px solid black; padding: 5px; display: inline-block;">\$</div>
		Box B

Make sure that the total in Box A is the same as the total in Box B. Please place this form in the envelope provided and wait silently for someone to come by and pick it up.

#### V

#### Room A

#### Follow-up Questionnaire

Why did you choose to allocate the money between your counterpart in Room C (person C) and your counterpart in Room D (person D) as you did?

**VI**  
**Room B**

**Results**

	<u>Number of envelopes correctly completed</u>	×	<u>Credit per envel.</u>	=	<u>Money credited to account</u>
Your counterpart (A)		×		=	\$
You (B)		×		=	\$
	_____				_____
<b>Total</b>					<b>\$</b>

Of this total, your counterpart decided to allocate the following amounts to:

Himself/Herself (A)		\$ _____
You (B)		\$ _____

**Questionnaire**

Consider the following situation: suppose you were your counterpart (person A). That is, suppose you prepared the same number of letters, got the same credit per letter and the same total credit to the account as A. Similarly, suppose you faced a counterpart (person B in Room B) whose letters prepared, per letter credit and total credit are the same as yours. Indicate how would you have allocated the total between

Person A		\$
Person B		\$
Total		\$

---

**VII**  
**Room C**

**Questionnaire**

Why did you choose to allocate the money between your counterpart in Room A (person A) and your counterpart in Room B (person B) as you did?

APPENDIX D: DATA

Benevolent/Discretionary			Standard/Discretionary				
	$x_a/\bar{x}$	$y_a/\bar{y}$	$\bar{y}$		$x_a/\bar{x}$	$y_a/\bar{y}$	$\bar{y}$
1	0.291667	0.433333	12.00	1	0	0	8.50
2	0.318182	0.318182	11.00	2	0.333333	0.833333	12.00
3	0.363636	0.5	11.00	3	0.384615	0.384615	13.00
4	0.380952	0.428571	10.50	4	0.407407	0.388889	13.50
5	0.4	0.4	10.00	5	0.416667	1.0	12.00
6	0.4	0.5	10.00	6	0.434783	0.434783	11.50
7	0.428571	0.224762	10.50	7	0.44	0.5	12.50
8	0.428571	0.428571	10.50	8	0.458333	1.0	12.00
9	0.44	0.32	12.50	9	0.47619	0.761905	10.50
10	0.44	0.4	12.50	10	0.5	0.5	12.00
11	0.44	0.44	12.50	11	0.5	1.0	11.00
12	0.473684	0.5	9.50	12	0.521739	0.5	11.50
13	0.473684	0.5	9.50	13	0.541667	0.5	12.00
14	0.48	0.4	12.50	14	0.541667	0.541667	12.00
15	0.5	0.5	10.00	15	0.545455	0.727273	11.00
16	0.545455	0.545455	11.00	16	0.565217	0.869565	11.50
17	0.565217	0.434783	11.50	17	0.583333	0.583333	12.00
18	0.565217	0.565217	11.50	18	0.583333	0.583333	12.00
19	0.578947	0.578947	9.50	19	0.590909	0.818182	11.00
20	0.619048	0.619048	10.50	20	0.608696	0.608696	11.50
21	0.636364	0.636364	11.00	21	0.636364	0.727273	11.00
22	0.636364	0.75	11.00	22	0.64	0.64	12.50
23	0.652174	0.695652	11.50	23	0.681818	0.681818	11.00
24	0.727273	0.727273	11.00	24	1.0	0.875	8.00

Benevolent/Exogenous			Standard/Double/Exogenous				
	$p_a$	$y_a / \bar{y}$		$p_a$	$y_a / \bar{y}$	$y_c / \bar{y}$	Type*
1	0.55	0.5	1	0.55	0.5	0.5	D
2	0.55	0.5	2	0.55	0.5	0.6	X
3	0.55	0.5	3	0.55	0.55	0.5	C
4	0.55	0.55	4	0.55	0.55	0.55	B
5	0.6	0.5	5	0.55	0.55	0.55	B
6	0.6	0.5	6	0.55	0.6	0.6	B
7	0.6	0.5	7	0.55	1.0	0.55	A
8	0.6	0.5	8	0.6	0.5	0.5	D
9	0.6	0.5	9	0.6	0.5	0.5	D
10	0.65	0.4	10	0.6	0.5	0.5	D
11	0.65	0.5	11	0.6	0.5	0.5	D
12	0.65	0.5	12	0.6	0.6	0.5	C
13	0.65	0.5	13	0.6	0.8	0.6	A
14	0.65	0.5	14	0.6	1.0	0.6	A
15	0.7	0.5	15	0.65	0	0.5	X
16	0.7	0.5	16	0.65	0.35	0.5	X
17	0.7	0.5	17	0.65	0.5	0.5	D
18	0.7	0.5	18	0.65	0.5	0.5	D
19	0.7	0.5	19	0.65	0.5	0.5	D
20	0.75	0.5	20	0.65	0.65	0.65	B
21	0.75	0.5	21	0.65	1.0	0.5	C
22	0.75	0.5	22	0.65	1.0	0.5	C
23	0.75	0.5	23	0.7	0.5	0.5	D
24	0.75	0.75	24	0.7	0.5	0.5	D
			25	0.7	0.5	0.5	D
			26	0.7	0.5	0.5	D
			27	0.7	0.5	0.7	X
			28	0.7	0.7	0.5	C
			29	0.7	0.7	0.7	B
			30	0.75	0.5	0.5	D
			31	0.75	0.5	0.5	D
			32	0.75	0.5	0.75	X
			33	0.75	0.6	0.6	B
			34	0.75	0.7	0.7	B
			35	0.75	0.7	0.7	B
			36	0.75	0.75	0.75	B

\*The subjects are identified as in Proposition 1 by type A, B, C or D. X indicates an outcome other than these four. These results are summarized as follows:

Type	A	B	C	D	X	Total
Number	3	9	5	14	5	36
Percentage of total	8.3	25.0	13.9	38.9	13.9	
Percentage of A-D	9.7	29.0	16.1	45.2		

Benevolent/Discretionary-Version 2

	$x_a/\bar{x}$	$y_a/\bar{y}$	$\bar{y}$
1	0.291667	0.291667	12.00
2	0.318182	0.318182	11.00
3	0.363636	0.363636	11.00
4	0.380952	0.380952	10.50
5	0.4	0.4	10.00
6	0.4	0.4	10.00
7	0.428571	0.428571	10.50
8	0.428571	0.476191	10.50
9	0.44	0.44	12.50
10	0.44	0.44	12.50
11	0.44	0.44	12.50
12	0.473684	0.5	9.50
13	0.473684	0.5	9.50
14	0.48	0.48	12.50
15	0.5	0.5	10.00
16	0.545455	0.545455	11.00
17	0.565217	0.565217	11.50
18	0.565217	0.565217	11.50
19	0.578947	0.578947	9.50
20	0.619048	0.619048	10.50
21	0.636364	0.636364	11.00
22	0.636364	0.681818	11.00
23	0.652174	0.652174	11.50
24	0.727273	0.772727	11.00