Chapter 5
A Marxist Model of Capitalist Growth

In Chapters 20 and 21 of volume II of Capital, Marx presents the necessary conditions for both simple and expanded reproduction of a capitalist economy. These conditions are first disturbed in the process of economic growth and then restored in the crisis. The nature of these conditions were never fully developed, however, nor were they completely integrated into the theory of capitalist accumulation. The roles of the rate of profit, fixed capital, and supply-side considerations (for example, the relation between accumulation and input markets) were never integrated with the conditions for expanded reproduction. In sum, the analyses of accumulation scattered through volumes I, II, and III of Capital were never unified in one story. It is possible to fill this gap by applying technical advances in growth theory since Marx’s pioneering efforts. We will do so in this chapter by developing a single-sector aggregate model. The conditions necessary for expanded reproduction—in modern terminology, equilibrium conditions for stable growth—will be analyzed. The disequilibrating over-investment tendencies described in the last chapter will be introduced into an equilibrium growth model. A hypothetical business cycle will be described. The underconsumption trap will be analyzed in greater depth. The rr curve used in the last chapter will be derived.

In section 1, a brief comparison of the important paradigms of growth theory will be made, followed by a prose description of the Marxist growth model presented in the later sections. Then, in sections 2 through 4, a mathematical model of capitalist growth will be presented. In section 5, the conclusions of this chapter will be summarized.
5.1 Alternative Theories of Growth and Distribution

In this section, a brief summary of the most popular theories of growth and distribution—the neoclassical and the von-Neumann-Sraffa—will be presented. In this summary, we will focus on two issues: first, the equilibration (or lack of equilibration) of the economy to a steady-state growth path, and, second, the determination of the rate of profit. The latter brings into focus the alternative theories of distribution. This summary and critique will be followed by a short prose exposition of the growth model to be presented in mathematical terms in the later sections of this chapter.

Harrod (1939) and Domar (1946) presented the first modern macroeconomic growth models (going beyond the work of Marx and Fel’dman). These theories dealt with growth, but not distribution: the rate of profit and the wage rate play no role whatsoever. In the Harrod-Domar model, the economy can grow at a constant rate at full capacity utilization with a constant unemployment rate. However, it is unlikely that it will do so. The economy acts as if it were balancing on a knife-edge, only avoiding a fall into depression or a surge into hyperinflation by accident. In fact, this model makes it seem unlikely that the economy actually survives for any length of time. Some mechanism for equilibration is needed, since the economy does not seem as unstable as the Harrod-Domar model suggests. One mechanism was suggested in Solow’s (1956) neoclassical growth model. At the same time, Solow introduced a theory of distribution. Let us turn to a consideration of this theory. (See chart 5.1 and Harris (1978, ch. 9) for a summary of this theory.)

It is useful to describe the neoclassical growth paradigm before it is criticized. As Gram notes:
5.1 The Neoclassical Growth Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Key Assumptions</th>
<th>Determined</th>
</tr>
</thead>
<tbody>
<tr>
<td>technology (production functions)</td>
<td>saving determines investment</td>
<td>distribution of income</td>
</tr>
<tr>
<td>population growth rate</td>
<td>full-capacity utilization</td>
<td>productivity</td>
</tr>
<tr>
<td>tastes (saving propensities)</td>
<td>equilibrium: employment rate and capital-labor ratio</td>
<td>rate of growth</td>
</tr>
</tbody>
</table>

5.2 The von Neumann–Sraffa Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Key Assumptions</th>
<th>Determined</th>
</tr>
</thead>
<tbody>
<tr>
<td>expectations, investment, rate of growth</td>
<td>full capacity utilization</td>
<td>distribution of income</td>
</tr>
<tr>
<td>technology (wage-profit frontier)</td>
<td>equilibrium: saving = investment ex post</td>
<td>productivity</td>
</tr>
<tr>
<td>tastes (saving propensities)</td>
<td></td>
<td>value of capital per worker</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rate of growth of employment</td>
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The neoclassical model ... is a miniature general equilibrium system of the Walrasian type. Such a model is distinguished by what it takes as given or parametric: resources and technology on the supply side (i.e., the factor ratio and the production functions ...) and preferences and factor ownership on the demand side (...). At each moment of time the model solves a static problem of allocating given means among alternative uses and, for that reason, prices have the interpretation as measures of relative scarcity and opportunity cost. (1976, p. 900.)

In this paradigm, it is quite common to confuse the rate of profit and the rate of interest (and at the same time, capital goods and capital funds or means of production and their value). In any event, the rate of profit (or interest) is seen as a price, determined by the scarcity and demand for "capital." The (in)famous "marginal product of capital" is simply a way of describing the demand for "capital."

In Solow's model, substitution of means of production for labor-power (or vice-versa) plays the role of equilibrating the system. This is combined with the assumption that some of the problems of the Harrod-Domar model are solved—Solow assumes that capacity is fully utilized and that saving determines investment (contrary to the conclusions of Keynesian analysis). The technology is assumed to be of the putty-putty sort. The problem remains that the rate of growth of employment must adjust to the rate of growth of the labor force. For Solow, substitution solves this problem: if the rate of growth of employment exceeds the rate of growth of the labor-force, means of production are substituted for labor-power, since the latter becomes more expensive. Thus, employment growth slackens and the economy converges toward an equilibrium growth rate where employment and the labor force are growing at the same rate so that the unemployment rate is constant.

However, this analysis—just as with Jorgenson's neoclassical theory of investment—assumes that the microeconomic process of factor
factor substitution is simply and directly reflected at the macroeconomic level. Consider a two-sector neoclassical growth model, with homogeneous machinery and substitution allowed in either or both sectors. Using this model we can negate Solow's analysis. Assume that total employment is growing faster than the labor force so that the wage increases. This rise might cause the substitution of machinery for labor-power at the microeconomic level, but if this occurs, there will be greater demand for labor in sector 1 (the machine-producing sector) unless there is strong substitution of machines for labor-power there. But this latter substitution also creates demand for labor-power in sector 1. Thus, it is unlikely that substitution will actually lead to an aggregate decline in the demand for labor-power. (Harris, 1973, calls this the "composition effect." Also, it is important not to ignore the costs to the working class of this process.) In fact, if sector 1 uses more labor-power per unit output than does sector 2 (after substitution occurs) then the aggregate demand for labor-power could actually rise in response to high wages. But if wages rise, means of production will become more expensive, discouraging any substitution. (This is roughly equivalent to Harris' "price effect.") So, from a rigorous analysis of a two-sector neoclassical growth model, Gram concludes that

an inverse relationship between the value of capital per man and the rate of profit [or a direct relationship between the value of capital per worker and the wage rate] is not a necessary property of the neoclassical growth model other than the one-sector model. (1976, p. 899.)

The same conclusion applies to output per worker and the real ratio of machinery per worker. We see these results because machines represent dead labor (that is, a series of labors done at different times in the past). Thus, ultimately there is no substitution for labor-power.
Instead, it is the time-path of the employment of labor-power that changes. Another implication of a two-sector model is that the rate of profit is not the price of "capital." There is a production price of machinery or "capital goods" ($p_1$) and a price of capital funds (the interest rate), but as we shall see, the rate of profit plays a different role.

Gram concludes that the neoclassical model is important if one wishes to describe optimal growth, that is, the world as it should be rather than as it is:

If capital theory is to focus primarily on problems of the efficient allocation of resources over time, then such dynamic generalizations of the Walrasian model will be the most appropriate vehicle of analysis. On the other hand, if the dynamic character of specifically capitalist economies is of primary interest, then models of the Marx-von Neumann-Sraffa type suggest themselves... (1976, p. 901.)

This conclusion about the neoclassical growth paradigm is independent of the "Cambridge Capital Criticism" (see below).

It is possible to do neoclassical analysis with a multi-sector model. For example, Solov's (1963, 1966) analysis assumes a world without an aggregate production function. He describes a planned economy that allegedly mirrors a perfect-market economy. He saw the rate of return as "entirely independent of the institutional arrangements of the economy" and not equal to "the rate of profit or the observed market rate of interest or any form or income in a capitalist economy." (1963, p. 17)

He concludes that

through all the vicissitudes of "normal" [that is, the single-sector case] and "pervasive" [reswitching and capital reversal] cases, however the rate of interest [here meaning the rate of profit] is actually determined, so long as full employment and competitive pricing prevail, the interest rate is an accurate measure of the social rate of return to saving. (1966, p. 30)

That is, in an economy where benevolent technocratic planners hold the economy in Golden Age equilibrium, the rate of profit equals the ratio
of future consumption gain to current sacrifice of consumption (the "social rate of return"). Thus, it seems that neoclassical growth theory applies best in an unknown utopia where the economy is driven to serve consumption. But there may be a snake even in this neoclassical Eden. Lancaster (1973) shows that if the economy is divided into classes (workers versus capitalists or workers versus bureaucrats) there is conflict over intertemporal consumption decisions, that is, over what is "optimal."

Since we are interested in studying the real world of capitalism, let us turn to the "Marx-von Neumann-Sraffa" model as Gram suggests. See chart 5.2 and Harris (1978, ch. 8) for a summary. Here also we substitution in production, but it is not as restricted in form as neoclassical substitution. Means of production need no longer be homogeneous, so we can see "reswitching" and "capital reversal." Capitalists choose between different "blueprints" of production coefficients in each sector. 7 (This is similar to the framework of the Okishio theorem discussed in the appendix to chapter 2.) More important here than the issue of changing techniques (and changing time paths of the employment of labor-power) is the different paradigm of the "Marx-von Neumann-Sraffa" model. This model

is best seen ... as a general equilibrium model of the classical type, ... The rate of accumulation [for Gram this means investment,] (determined by the owners of capital), savings propensities (identified with a social division between labor and capital in production), and the available technology, govern the process of accumulation and the distribution of income. Equilibrium prices are determined by the requirement that the rate of profit be equalized in each sector, and not by the requirements of a momentary equilibrium of supply and demand, since resources that are reproducible means of production are not parameters of the problem. (Gram, 1976, p. 900-1.)
The von Neumann-Sraffa model combines classical and Keynesian assumptions in a novel way. The classical assumption of full capacity utilization is made, but, following Keynes, investment (exogenously given by "animal spirits" or whatever) determines saving ex post. Because saving rates and the distribution of income are closely related, investment thus determines the rate of profit. Given this rate of profit, the real wage rate is determined subject to the constraint that the economy is on the wage-profit (or "factor-price") frontier. The latter is the full capacity relationship between the rate of profit and the wage rate, determined by the technical coefficients of the economy (the book of blue-prints) and the assumption that the rate of profit and the wage rate are each equilized among sectors. The rate of profit is not a price as in the neoclassical model. Rather, it is determined by the rate of saving, which has to adjust to equal investment. The rate of profit might be described as a "buffer" that allows the economy to adjust to equilibrium, though some have given it a Marxian interpretation.

While the von Neumann-Sraffa model is more accurate that the neoclassical theory in its theory of prices and technology, it returns us to the Harrod-Domar knife-edge. There is nothing to assure us that the actual rate of growth will equal the "natural" rate of growth (and that employment will grow with the labor-force). The economy suffers from disequilibrating forces without any countervailing equilibrating tendencies. This is the reverse of the neoclassical model with its equilibrating tendencies and no endogenous disequilibration.

Gom lists Marx along with von Neuman and Sraffa as applying this paradigm. While this is roughly accurate for the theory of equilibrium prices of production, it is not so for the theory of growth. The above
theory should perhaps be labelled the von Neumann-Strafa-Kaldor (or Cambridge) theory. After all, it was N. Kaldor (1955-6) who emphasized the theory of differential saving propensities between classes and ex post equality of aggregate saving and (exogenously given) investment as determining the distribution of income in full-capacity growth.

Not only is the Cambridge theory not that of Marx, but Marx's general conception is more realistic. First, Marx saw capitalist growth as not simply a set of simultaneous equations representing economic expansion, but also a broad socio-economic process of the reproduction of the capitalist mode of production on an ever-widening scale. (See chapter 3.) Thus, the following mathematical construction should be seen as only one part of a more complex dynamic process. Second, for Marx the rate of investment is dependent on the rate of profit rather than being simply "determined by the owners of capital." (Strictly speaking, Marx never distinguished systematically between saving and investment. But this distinction is an innovation it seems foolish to reject.) For Marx, the rate of profit was not a price—or a buffer—but a result of capitalist ownership of the means of production (and workers' non-ownership of them) and their resultant control over saving and investment decisions. Profit is thus a deduction from the product of labor. Third, as Harcourt notes a world of Golden Ages, of realized expectations, and of rigorously defined measures of 'capital' is not the natural habitat of a practicing Marxist. (1973, p. 1267)

As was argued in the introduction to chapter 2, Marx was concerned with dynamic disequilibrium and disequilibrium.

In some ways, Marx's work—and the Marxian model of capitalist growth to be described below—are similar to the work of Robinson (1962, 1969) and the post-Keynesian school. Her books and articles describe
growth out of equilibrium where both saving and investment are determined by the rate of profit. Nonetheless, Robinson's work is in the Keynesian tradition, emphasizing growth below full capacity and the effects of aggregate demand while de-emphasizing supply-side elements. Her model is just part of the story.

Marx's method of analysis suggests a solution to the Harrod-Domar knife-edge. If, for example, employment is growing faster than the labor force, this will eventually lead to a rise in the general wage rate and (all else equal) cause a fall in the rate of profit. This implies a cut-back in both investment and saving. Thus, a fall in the rate of profit forcibly equilibrates the system, by cutting back employment growth. After the crisis, however, it is likely that employment will be growing more slowly than the labor force. This must occur until the wage rate is depressed enough to restore the rate of profit, saving and investment. (Goodwin, 1972, and Harris, 1978, ch. 10, capture this dynamic.) Thus, we see an interplay between equilibrating and disequilibrating forces. For the Marxian model presented here, the disequilibrating forces (in what I label the "medium term") are the knife-edge problem and tendencies toward induced investment (as described in chapter 4), while the equilibrating forces are those that change the rate of profit to push the economy toward equilibrium. These are supply-side factors and adjustments in the capacity-capital ratio: if the actual rate of growth deviates from the natural rate of growth and the unemployment rate is not equal to the medium-term equilibrium rate, these factors can affect the rate of profit to move the economy toward equilibrium.

In some ways, this adjustment mechanism is similar to the neoclassical mechanism suggested by Solow (1956). The argument against his aggregate
production function suggests that the technologically-determined aggregate demand for labor-power has zero elasticity with respect to a general change in the wage rate. But the Marxian mechanism discussed in the previous paragraph implies that the aggregate demand for labor-power is elastic with respect to a general change in wages since as wages rise, the rates of profit and investment fall. But this elasticity of the demand for labor-power is socially determined, that is, part of capitalism as a social system. It is based on the capitalists' control of the accumulation process and their ability to punish society if they are not pleased with their lot. Neoclassical theory takes this social phenomenon and makes it part of the technology—making the dynamics of capitalism seem part of Nature. Since neoclassicals are most concerned with equilibrium states, this is an especially harmonious world.

The two- (or many-) sector model will remain implicit in the analysis. That is, the substitution of machinery for labor-power, the shift of demand between sectors, and relative price changes will be of secondary importance. Movements of the economy will be mostly below or above the wage-profit frontier. The model is macroeconomic, focusing on the saving and investment processes and the aggregate rate of profit. We will be considering a representative firm that is an average of all the firms in the economy (just as for the formula for the rate of profit in chapter 3).

The model presented below could be stated in terms of distributed lags and a deterministic model. Such would be quite the opposite of the purpose of the model, however. Hence only equilibrium conditions and the general processes of equilibration and disequilibrium will be described. The role of lags is approximated by the different speeds of adjustment of
the different variables. These speeds are approximated by the threefold distinction among the short, medium, and long terms. The short term represents a single phase (or moment) of the business cycle, during which costs are constant and aggregate demand adjusts. The medium term corresponds to the period when costs adjust, a larger segment of the 4 to 10 year business cycle. The long term sets the context for the medium-term fluctuations of the economy—it is here that trend rates of growth of the labor force, productivity, and so forth are determined. But the long term is not independent of the medium term since (as we shall see) the series of medium term periods determine the trend. The long term does not represent a notional Golden Age that could be examined independent of the cycle. Capitalist growth is essentially cyclic; cycles are not secondary fluctuations in the context of a secular trend determined by Nature.

The short term corresponds roughly to the short-term equilibria of the simply cycle model at the end of the last chapter. However, it will not be assumed that expectations are in equilibrium in the short term. Instead, the investment function where expectations are left implicit (equation 4.9) will be used. The medium term corresponds to the phase of the analysis where the equilibria begin to migrate; here, expectations-adjustment is part of the adjustment. The long-term considerations determine the general location of the curves.

For simplicity, it will be assumed that all raw materials are imported so that a fall in the terms of trade will imply a shift of surplus out of the country. Inflation will be assumed away. One way to do this is to return to Marx's simplifying assumption that money is directly based on gold, which is supplied elastically at a constant price of production to serve the needs of trade. We could instead assume that
foreign products market is intense, the exchange rate is constant, and there is no inflation abroad.

In the short term, the model is a simple dynamized Keynesian one. The rate of growth warranted by investment \( (G_t) \) and the rate of growth warranted by saving \( (S_t) \) are determined independently (though they depend on similar variables). They thus need not be equal at any given rate of capacity utilization. The economy adjusts so that planned saving = planned investment in short-term equilibrium. In the static Keynesian model, it is national income that adjusts. Here, the key variable that adjusts is the rate of capacity utilization \( (z) \). To some extent, the general price level \( (p) \) also adjusts. A rise in the rate of capacity utilization raises the rate of profit and encourages investment while at the same time, saving rises because the share of profits rises as overhead costs play a smaller role in costs. Thus, saving and investment can adjust to equalize and so determine the short-term equilibrium rates of capacity utilization and profit. In the short-term, the rate of profit acts as a buffer, just as in Cambridge models. (However, unlike some Cambridge models, the way in which profits are adjusted to allow equilibrium is clear.) Stability conditions for short-term equilibrium will be investigated.

Short-term equilibrium need not be a medium-term equilibrium. Medium term equilibrium is defined as the situation where labor-power markets, raw-material markets, and expectations are in equilibrium while the actual rate of growth equals the "natural" rate of growth. The measure of the share of profits independent of the rate of capacity utilization (the gross margin, \( m \)), the terms of trade, and the capacity-
capital ratio are constant. The economy is at the medium-term equilibrium rate of capacity utilization ($s^*$) and the medium-term equilibrium rate of profit ($r^*$) is determined.

This theory is akin to that of Solow and Stiglitz (1968) in that the model is similar to the post-Keynesian model in disequilibrium (that is, in short-term equilibrium) but is not so in medium-term equilibrium. However, the determination of the distribution of income is distinctly non-neoclassical since there is no aggregate production function and the economy is not presumed to serve consumption.

Also, medium-term equilibrium is unstable because of disequilibrating over-investment tendencies. After a short-term period where the medium-term equilibrium conditions are met, the economy will move out of equilibrium, so that employment will no longer grow with the labor force and the rate of capacity utilization no longer equals $s^*$. Because of the instability of medium-term equilibrium, medium-term disequilibrium dynamics are very important. Consider, for example, a situation where employment is growing faster than the labor force and the unemployment rate is low, while the demand for raw materials is growing faster than supply and there is excess demand. Since there is (by assumption) no inflation, this situation leads to a fall in capital's share at any given rate of capacity utilization (a fall in $s$), plus a fall in the terms of trade ($F$). Simultaneously, the capacity-capital ratio ($h$) falls because the stock of means of production increases faster than does output per machine in the medium term. Also, it is possible that $h$ falls because of bottlenecks in production. These forces cause a fall in the rate of profit

$$r = f(s, m, F) h$$

(2.14')
when the falls of $u$, $h$, and $P$ swamp the rise of $z$ in the economic boom. (I will call these first three variables the vector $a$.) The fall in the actual rate of profit will eventually mean a fall in the expected rate of profit ($r^e$). Thus, investment and saving will fall. The rate of profit is restored at the expense of the working class by increasing the size of the pool of unemployed workers and at the expense of raw-material sellers by increasing the terms of trade.

However, the rate of profit does not recover immediately. In fact, it will fall in the short term, since $z$ will fall in an economic recession. Also, imbalances—low values of $u$, $P$, and $h$ associated with a relatively strong working class, raw material sellers' cartels, and over-investment in fixed capital—may develop in the boom. These imbalances have to be purged from the system in order to restore the rate of profit. For example, an extended period of massive unemployment and/or a wave of union-busting will raise $u$ if there is little popular resistance. A long period of under-investment will raise $h$. However, it is possible that the economy will get stuck in an underconsumption trap.

There are several different types of medium-term disequilibria. These will be described in detail. It is here that business cycles, the underconsumption trap, price dynamics, and the derivation of the $rr$ curve are discussed.

While in the short term the rate of profit is determined by the rate of capacity utilization (and aggregate demand), in the medium term it is determined more on the supply-side. It is determined by the relationship between the rate of growth of the economy and supply considerations. In the long term, the basic parameters of the system, especially the trend rate of growth of productivity, the labor force,
the supply of raw materials, and wages, and determined. See diagram 5.3 for a summary of the relationships between the different time frames.

This chapter will use the following symbols: for a variable \( x \),
\[ \dot{x} \]
means \( \frac{dx}{dt} / x \), the percentage rate of growth of \( x \) over time; \[ \ddot{x} \]
means the trend rate of growth of \( x \); \[ \bar{x} \]
means the trend value of \( x \). (Thus, \[ \ddot{x} = \ddot{x} \].) Time subscripts will be left out in most cases.

The short term, the medium term, and the long term will be considered in turn, in sections 5.2, 5.3, and 5.4. Section 5.3, the longest section, will be divided between those sections describing medium-term equilibrium (sections 5.3.1 and 5.3.2), those describing the determination of the elements of \( g \) (sections 5.3.3-5), and that describing disequilibrium dynamics (section 5.3.6).
5.3. The Marxian Growth Model.

**Short-Term Equilibrium**

Medium-Term Considerations

Equilibrium: \( G_I = G_S \).

**Medium-Term Dynamics**

Long-Term Considerations

Short-Term Equilibrium