The exam will consist of two parts, both of which you will receive at the beginning of the period and you can work on both of them at the start. You may use your cheat sheet during the entire exam. However, you are not to use your calculator on Part 1. You may hand in Part 1 at any point, after which you are permitted to pull out your calculator and use for Part 2 (which you could have been working on from the beginning without a calculator).

Here is my advice to you:

(a) Do as much of both parts of the exam as you can without the calculator.

(b) Next, go back and look at Part 1 and answer any questions you skipped.

(c) Hand in Part 1 and pull out your calculator to complete Part 2.

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1. Exercises 23, 29, 33, 35 from Section 3.9 on pp. 207–208.

2. Exercises 13, 43, 51, 57, 61 from Section 3.10 on pp. 218–219.

3. Exercises 45, 51, 53 from Section 4.1 on p. 244.

4. Exercises 5, 13, 29, 39, 49 from Section 4.3 on pp. 258–259.

5. Exercises 19, 21, 29 from Section 4.4 on p. 268.

6. For each of the following, calculate the derivative of y with respect to x.

(a) $3y^3 + x^2 = 5$

(b) $x^2y + 2xy^2 = x + y$

(c) $\sqrt{x+y} = \frac{1}{x} + \frac{1}{y}$

(d) $y + \frac{x}{y} = 1$

(e) $\sin(x + y) = x + \cos y$

Answer: (a) $-(2x)/(9y^2)$, (b) $(1 - 2xy - 2y^2)/(x^2 + 4xy - 1)$, (c) $-(y^2(x^2 + 2\sqrt{x+y}))/(x^2(y^2 + 2\sqrt{x+y}))$

(d) $y/(x - y^2)$, (e) $(1 - \cos(x + y))/(\cos(x + y) + \sin y)$

7. Find $\frac{dy}{dx}$ at the point $(1, -1)$ on the curve

$$(x + 2)^2 - 6(2y + 3)^2 = 3.$$

Answer: 1/4

8. For each of the following, find an equation of the tangent line at the given point.

(a) $xy - 2y = 1, \quad (3, 1)$

(b) $x^{2/3} + y^{2/3} = 2, \quad (1, 1)$

Answer: (a) $y = 4 - x$, (b) $y = 2 - x$

9. Find the points on the graph of $y^2 = x^3 - 3x + 1$ where the tangent line is horizontal.

Answer: $(-1, \sqrt{3}), (-1, -\sqrt{3})$

10. Show that no point on the graph of $x^2 - 3xy + y^2 = 1$ has a horizontal tangent line.

11. Find equations of the tangent lines at the points where $x = 1$ on the graph of

$$(x^2 + y^2)^2 = \frac{25}{4}xy^2.$$

Answer: $(1, 2) : y = (1/3)(x - 1) + 2,\ \ (1, -2) : y = -(1/3)(x - 1) - 2$

$(1, 1/2) : y = (11/12)(x - 1) + (1/2),\ \ (1, -1/2) : y = -(11/12)(x - 1) - (1/2)$

12. Show that if f and g are differentiable, then

$$\frac{d}{dx}\ln(f(x)g(x)) = \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)}.$$

13. Use logarithmic differentiation to compute the derivative of

$$y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}.$$

Answer: $y' = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right).$

14. Use logarithmic differentiation to compute y' for each of the following:

(a) $y = x^{2x}$

(b) $y = x^{(e^x)}$

(c) $y = x^{(2^x)}$

Answer: (a) $y' = x^{2x}(2 + 2\ln x)$, (b) $x^{(e^x)}(\frac{e^x}{x} + e^x \ln x)$, (c) $x^{(2^x)}(\frac{2^x}{x} + 2^x \ln 2 \cdot \ln x)$

15. Find an equation of the tangent line of the function at the indicated location:

(a) $f(x) = 5^{x^2-2x+9}$ when $x = 1$.

(b) $g(x) = 4\ln(9x + 2)$ when $x = 2$.

(c) $h(x) = \log_5(x)$ when $x = 2$.

Answer: (a) $y = 5^8$, (b) $y = (9/5)$

16. Let $f(x) = x^2 - 4x - 5, x > 2$. Find the value of $\frac{df^{-1}}{dx}$ the the point $x = 0 = f(5)$. Draw a picture of this scenario. Answer: 1/6

17. Compute the derivatives of the following functions.

(a) $f(x) = \ln(\ln(\ln x))$

(b) $g(t) = \log_{10}(t^3 + t^2)$

(c) $h(x) = 2^{(x^3)}$

(d) $f(x) = \tan^{-1}(\ln(x))$

(e) $h(x) = \sin^{-1}(\cos x)$

(f) $r(\theta) = \theta^2 \cos^{-1}(\theta^2)$

Answer: (a) $1/(x(\ln x)(\ln(\ln x)))$, (b) $(2t+3t^2)/((t^2+t^3)\ln 10)$, (c) $3(\ln 2)x^2 \cdot 2^{(x^3)}$, (d) $1/(1+\ln(x)^2)$

(e) $-(\sin\theta)/(\sqrt{1-\cos^2\theta})$, (f) $2\theta\cos^{-1}(\theta^2) - \frac{2\theta^3}{\sqrt{1-\theta^4}}$

18. A rocket that is launched vertically is tracked by a radar station located on the ground 3 miles from the launch site. What is the vertical speed of the rocket at the instant its distance from the radar station is 5 miles and the distance is increasing at the rate of 500 miles per hour?

Answer: 6250 miles per hour

19. A man six feet tall walks with a speed of eight feet per second away from a street light that is atop a 18 foot pole. How fast is the tip of his shadow moving along the ground when he is 100 feet from the light pole?

Answer: 12 feet per second

20. Two radar stations at A and B, with B 6km east of A, are tracking a ship. At a certain instant, the ship is 5km from A, and this distance is increasing at a rate of 28 km/hr. At the same instant, the ship is also 5 km from B, but this distance is increasing at only 4 km/hr. Where is the ship, how fast is it moving, and in what direction is it moving?

Answer: 3km east and 4km north of A (or 3km east and 4km south of A), $20\sqrt{2}$ km/hr northeast

21. Find the maximum and minimum values of the given function on the given interval.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 15$ on $[0,3]$

(b) $g(x) = 3 - |x-2|$ on $[1,4]$

(c) $f(x) = 5x^{2/3} + x^{5/3}$ on $[-1,4]$

(d) $f(t) = \frac{1}{t} + \ln t$ on $[0.5, 4]$

(e) $h(t) = e^{-x^2}$ on $[-2,1]$

Answer: (a) $f(0) = 15$, max, $f(2) = -5$, min, (b) $f(2) = 3$, max, $f(4) = 1$, min, (c) $f(-1) = 6$, max,

$f(0) = 0$, min, (d) $f(4) = (1/4) + \ln 4$, max, $f(1) = 1$, min, (e) $f(0) = 1$, max, $f(-2) = e^{-4}$, min

22. Show that the function $f(t) = \frac{1}{1-t} + \sqrt{1+t} - 3.1$ has exactly one zero in the interval $(-1,1)$.