Math 132: Calculus II, Spring 2006, Section 01, CRV 71224

**Typo Corrected Project 02. Due 2006 April 21 at 9:00am PDT.**

This project relates to the integrator feedback controller

\[ a_{n+1} = a_n + ge_n. \]  

(1C)

1. Show that \( a_{N+1} = a_1 + g \sum_{n=1}^{N} e_n \).

2. Explain how the equation in (1) is related to the Fundamental Theorem of Calculus on Page 309. Explain how the equation (IC) is related to the definition of derivative on Page 115.

3. Often a modeling assumption for the application of (IC) is that the error signal \( e_n \) comprises the control action and an external disturbance: \( e_n = w_n - a_n \). Show that for this model, the IC equation becomes

\[ a_{n+1} = (1 - g)a_n + gw_n. \]  

(1CM)

4. Find a summation equation for \( a_{N+1} \) that depends only on \( a_1 \) and the \( w \) sequence.

5. Apply the integrator model (ICM) to the three experiments in the excel spreadsheet project_02.xls downloadable from

http://myweb.lmu.edu/bfitzpatrick/Math132.htm

Use \( g = 0.1, 0.5, \) and 0.99.

6. In most applications, these signals are sampled: \( w_n = x(n \Delta t), \ a_n = y(n \Delta t) \), from a continuous time process in operation. If \( g = r \Delta t \), plug these sampled functions into the equation (ICM) and let \( \Delta t \to 0 \). Show that this process leads to the equation

\[ \frac{dy}{dt} = -ry(t) + rx(t). \]  

(CICM)

7. Use the Fundamental Theorem of Calculus to show that this equation is solved by the formula (corrected version is below, contains a factor of \( r \) not in old one)

\[ y(t) = e^{-rt} y_0 + e^{-rt} \int_0^t e^{rs} rx(s)ds \]
8. Use the integrator (IC) to control your experiment, which is to be conducted during class time Tuesday 2006 April 18.

**Hints.** Problem 1: Compute $a_1, a_2, a_3$ and look for the pattern. OR, even better, start with the summation formula and compute $a_{N+1} - a_N$.

Problem 2. Jump ahead to Problems 6 and 7, or think about the “even better” hint above.

Problem 3. This one is easy. Don’t make it hard.

Problem 4. Compute $a_1, a_2, a_3, a_4$ and look for a pattern


Problem 6. The definition of derivative on Page 115. That 1 in 1-g is a friend of the left hand side.

Problem 7. Product rule plus the FT0’C. Plug and Chug.

Problem 8. Fun city. Wear comfortable clothes on Tuesday.

**Rules.** You may not discuss your work with anyone outside your team except Dr. Fitzpatrick. You will need to schedule an appointment to use my thermometer if you do not have your own. Papers are due at 9am on Friday, 2006 April 21. Late papers will not be accepted under any circumstances.

**The following are the team assignments for this project.**

- Brown: Buljan, Espino, Tingzon.
- Blue: Evans, Hinckson, McKormick.
- Black: Khalaf, Shribbs, Somma.
- Orange: Diaz, Novoa, Geren.
- Blonde: Christensen, Mendez, Naphtal.
- Pink: Lim, Harmon, Perez.