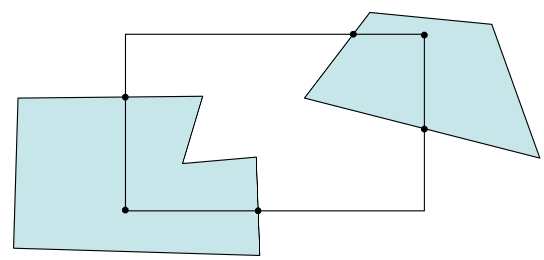
Clipping

- Remember, we are in implementation mode now, and so we need to worry about not drawing things out of bounds — a process called *clipping*
- Clipping isn't just deleting points that are outside our desired bounds; it also includes calculating where shapes *intersect* the bounds, since that's what we'll actually draw



Clipping Algorithms

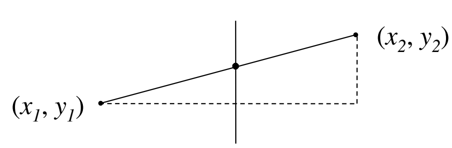
- Many clipping algorithms — in pure form, they are independent of number of dimensions, but for simplicity we will look at the 2D versions
- General criteria for a good clipping algorithm:
  - Quickly identify lines (or polygon edges) that need not be drawn at all: these can be skipped and discarded
  - Quickly identify lines (or polygon edges) that are completely contained within the drawing bounds: these can be drawn without modification
  - For the lines that are neither (i.e., they cross display boundaries), calculate their intersections as quickly as possible: the less of these we do, the better

Cohen-Sutherland Clipping

Translate boundary crossings into bit fields; for example, in 2D this is 4 bits, one each to represent in/out the top, bottom, right, or left boundaries (0 = in bounds, 1 = out of bounds)

1001	1000	1010
0001	0000	0010
0101	0100	0110

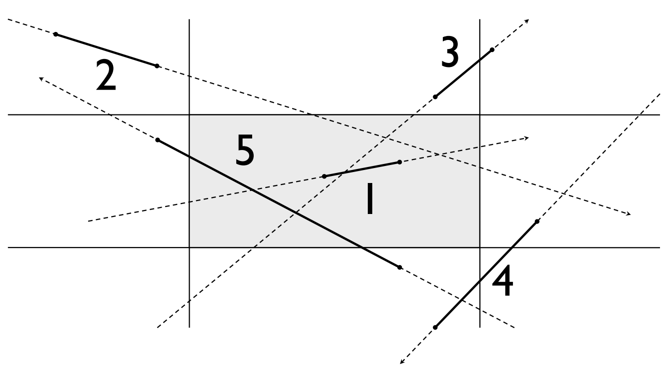
- For every vertex (x, y), initially assign 0000
- If $x > R$ then set bit 1 else if $x < L$ then set bit 0
- If $y > T$ then set bit 3 else if $y < B$ then set bit 2
  - Note how when $x = R$ or $x = L$, or $y = T$ or $y = B$, the corresponding bit is still zero; equality = in bounds
- Now, for every line segment (x_1, y_1) to (x_2, y_2):
  - If both line segments are 0000, then draw line; we're done (*accept*)
  - If $code_1$ & $code_2$ (bitwise *and*), then skip line; we're done (*reject*)
  - Otherwise, calculate the intersection with a boundary; re-encode then reconsider this new pair of endpoints



Liang-Barsky Clipping

- Remember linear interpolation? It allows us to write a line in terms of a parameter *u* from 0.0 to 1.0: $x = x_1 + u(x_2 - x_1)$, $y = y_1 + u(y_2 - y_1)$
- Liang-Barsky asks: for what values of *u* does a line segment enter or exit the bounds?
- There can be, at most, two of each; we care about the maximum entry value and the minimum exit value

- For each line segment, for each boundary, check the value of *u* at the intersection of the segment's line with that boundary
- If $u < 0$ on entry and $u > 1$ on exit — *accept*
- If $u > 1$ on entry or $u < 0$ on exit — *reject*
- If *u* on entry > *u* on exit — *reject*
- Otherwise, clip and try again — note how we don't need to perform an extra calculation, because the new point can be derived from *u*



Line 1: max entry < 0, min exit > 1 — *accept*

Line 2: max entry > 1, min exit > 1 — *reject*

Line 3: max entry < 0, min exit < 0 — *reject*

Line 4: max entry > min exit — *reject*

Line 5: max entry > 0, min exit < 1, max entry < min exit — *clip*

Liang-Barsky Algorithm

- For a given line segment (x_1, y_1) to (x_2, y_2), derive the parametric form of its line: $x = x_1 + u(x_2 - x_1)$, $y = y_1 + u(y_2 - y_1)$
- For each boundary (L, R, T, B), calculate the value of u for that line at that boundary; note that a point is within the boundary if:

$$L \leq x \leq R \text{ and } B \leq y \leq T$$

- Substituting the parametric form, let:

$$dx = x_2 - x_1, dy = y_2 - y_1$$

$$L \leq x_1 + u(dx) \leq R \text{ and } B \leq y_1 + u(dy) \leq T$$

- If we break these inequalities up, we get these conditions:

$$-dx\ (u) \leq x_1 - L \rightarrow \text{let } C = -dx, q = x_1 - L$$

$$dx\ (u) \leq R - x_1 \rightarrow \text{let } C = dx, q = R - x_1$$

$$-dy\ (u) \leq y_1 - B \rightarrow \text{let } C = -dy, q = y_1 - B$$

$$dy\ (u) \leq T - y_1 \rightarrow \text{let } C = dy, q = T - y_1$$

- Note how, for each C and its corresponding boundary:

  $C < 0 \Rightarrow$ line goes out → in: *entry*

  $C > 0 \Rightarrow$ line goes in → out: *exit*

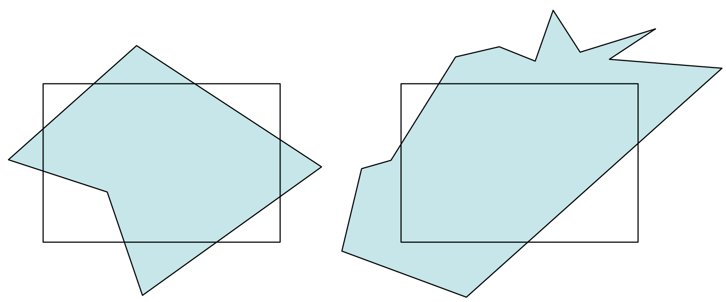
  $C = 0 \Rightarrow$ line is parallel to the boundary

- So, we can calculate u for each boundary by calculating q and C; the value of C tells us if we are looking at an entry or exit point for the boundary. Thus, we can now apply the conditions:
  - If $u < 0$ on entry and $u > 1$ on exit — *accept*
  - If $u > 1$ on entry or $u < 0$ on exit — *reject*
  - If u on entry > u on exit — *reject*

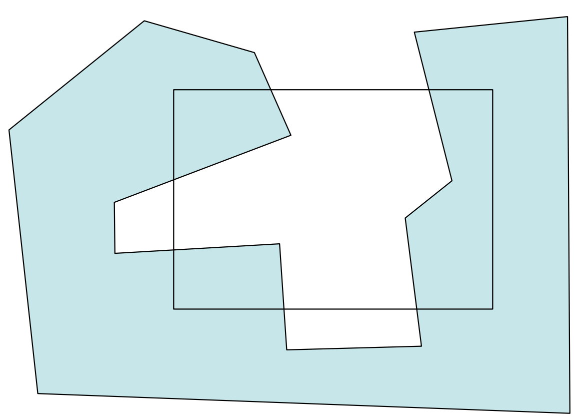
```
procedure ClipAndDrawLine(x1, y1, x2, y2: real) is
    u1: real := 0.0;    dx: real := x2 - x1;
    u2: real := 1.0;    dy: real := y2 - y1;

    function Reject(C, q: real) return boolean is
        u: real := q / C;
    begin
        if C < 0 then
            if u > u2 then return true; elsif u > u1 then u1 := u; end if;
        elsif C > 0 then
            if u < u1 then return true; elsif u > u2 then u2 := u; end if;
        else
            if q < 0 then return true;
        end if;
        return false;
    end Reject;

begin
    if Reject(-dx, x1 - L) then return; end if;
    if Reject(dx, R - x1) then return; end if;
    if Reject(-dy, y1 - B) then return; end if;
    if Reject(dy, T - y1) then return; end if;
    if u2 < 1.0 then (x2, y2) := (x1 + u2 * dx, y1 + u2 * dy); end if;
    if u1 > 0.0 then (x1, y1) := (x1 + u1 * dx, y1 + u1 * dy); end if;
    DrawLine(x1, y1, x2, y2);
end ClipAndDrawLine;
```

How About Polygons?

- Clipping polygons complicates the problem because we have to take account the enclosed area
- Also, where line segment clipping always maps two points to two (possibly other) points, polygon clipping may actually change the number of points after clipping

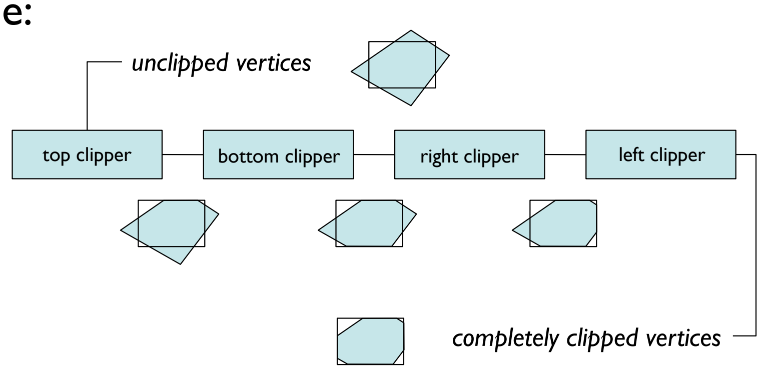


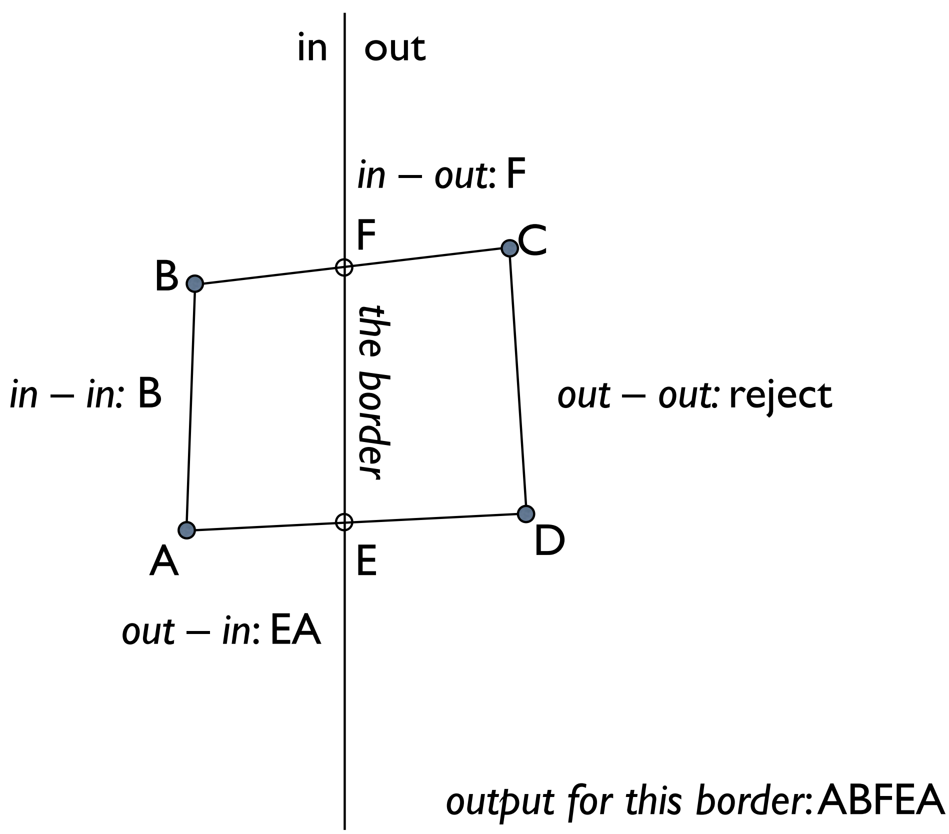
- In the ultimate extreme case, we may even end up with more than one polygon after clipping!

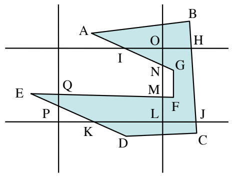


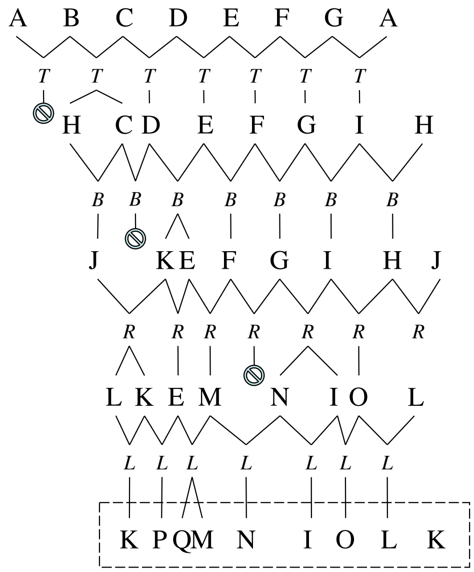
Sutherland-Hodgman Polygon Clipping

If we consider a clipping algorithm to be a black box that takes a set of vertices then produces a new set of clipped vertices, then without loss of generality we can clip along one border at a time and string them together — like a pipeline:









How About 3D?

- All algorithms extrapolate to 3D:
  - Cohen-Sutherland: add inside/outside bits for z axis
  - Liang-Barsky: add parametric equation for z coordinate
  - Sutherland-Hodgman: add pipes for near and far clipping
- Remember: clip in 3D, *then* project
  - One of the reasons why the projection matrix isn't multiplied immediately with the modelview matrix
  - Otherwise, you'll end up projecting points to the near plane that shouldn't be projected
  - Worse yet, you'll end up projecting points that are *behind* the camera and the near plane — that will actually result in an erroneous display!