Study Guide for test 2

The test will cover the material through section 3.10, with most of the questions being material from sections 2.7 to section 3.10. Overall, the emphasis of the test will be on taking derivatives, relating the graph of the derivative to the graph of the function, the derivative rules, logarithmic differentiation and related rates. In particular, you should be able to

- State and use the (limit) definition of the derivative.
- Understand the derivative as a rate of change.
- Be able to use the graph of the derivative to find information about the graph of a function.
- Solve simple problems involving derivatives, both by rules and by the definition.
- Understand the differentiation rules (product, quotient, and chain) and when to use them.
- Be able to evaluate the derivative of exponential functions and thesine function using the definition and the basic limits of $\frac{\sin(x)}{x}$, $\frac{\cos(x)-1}{x}$, and $\frac{e^x-1}{x}$ as x tends to 0.
- Know all trigonometric, exponential, and logarithmic derivatives.
- Do implicit differentiation and use it to find the derivative of the inverse trigonometric functions.
- To use logarithmic differentiation to find derivatives.
- Exponential Growth and Decay
- Solve related rates questions.

The test will most likely consist of two sections. A calculator (and formula sheet) free section and a calculator (and formula sheet) allowed section. You will be handed out both sections, but you may not take out your calculator (or the study sheet) until you have handed in the calculator free section. Overall, the test will contain both computational problems and problems involving analysis and conceptual understanding. There will be a related rates problem on this test. On the true-false portion of the test, you may explain your reasoning. Such an explanation may be worth some points if you have the

wrong answer but the reasoning is mathematically correct. No points will be taken off for incorrect reasoning if the answer is correct (though I may correct the reasoning when grading your test). The test will also likely contain a problem requiring you to explain in your own words something from class. This answer should be written in complete sentences and be appropriate for a student just starting in calculus. For this test you may compile a single page of formulas for the calculator allowed section of the test. This page may consist of anything you consider useful, but it may only be a single page.

On the next pages, I have included a sample test. This is a sample of the type of test I give, but the problems on the actual test may be substantially different from those on the sample test.

Name= Sample Test

Part 1
no calculator or formula sheet allowed

1. Compute $f'(x)$ for each of the following functions.

a. $f(x) = x^3 + \sin(x)$

b. $f(x) = \cos(2x)\ln(x^2 + 1)$

c. $f(x) = 3^{(x^7+5x)^3}$

d. $f(x) = \log_5(x\sin(x))$

e. $f(x) = \ln(7e^2)$

f. $f(x) = \frac{\tan(x^2)}{(7x^3+2x+1)}$

g. $f(x) = (x^3 + 2x + 1)^{10}$

2. Write the equation of the line tangent to the curve $y^3x = 3x^2 - 10$ at the point $(2, 1)$.

Part 2
calculator and formula sheet allowed

3. Use the definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{x}$.
4. Use implicit differentiation to find the derivative of $\text{Arctan}(x)$.
5. True or False. Mark each statement as true (True) or false (False) (please write the entire word). Partial credit may be given for good explanations even if the answer is incorrect.

☐ If f and g are both differentiable, then

$$\frac{d}{dx}f(g(x)) = f'(g'(x)).$$

☐ The derivative of the function $f(x)$ at $x = a$ is the line tangent to $f(x)$ at the point $(a, f(a))$.

☐ Since the derivative of a constant is always 0, if c is a constant then $\frac{d}{dx}cf(x) = 0$.

☐ If f is a differentiable function, then the second derivative of f must exist.

6. Let $f(x)$ and $g(x)$ be differentiable functions with the following values $f(2) = 6$, $f'(2) = 1$, $f(4) = -3$, $f'(4) = 2$, $f(5) = 3$, $f'(5) = 10$, $g(2) = 4$, $g'(2) = 5$, $g(4) = 5$, $g'(4) = 4$, $g(5) = 2$, and $g'(5) = 5$. Find

a. The derivative of $f(g(x))$ at $x = 4$.

b. $(fg)'(2)$.

c. The derivative of $\frac{f(x)}{g(x)}$ at $x = 5$.

7. Suppose $f(x)$ is a function with $f(10) = 15$ and $f'(10) = \frac{1}{2}$. Use linear approximation to find an approximate value for $f(10.3)$. (Extra Credit: If $f'(x) \leq 3$ for all x, for what values of x can you guarantee that the linear approximation for $f(x)$ based at $x = 10$ is within .5 of the correct answer?)

8. Princess Leia, Han Solo, Chewbaca, and Luke Skywalker are trapped in a trash compactor that is 20 feet long. The cross section of the trash compactor is an isosceles triangle. The two walls of the trash compactor are both 20 feet long, and slide along the floor (causing the height of the join of these two walls to go up). The base of the walls are coming together at the rate of 2 feet per second. At what rate is the volume of the trash compactor decreasing when the walls are 5 feet apart at the base?

9. A sample of tritium-3 decayed to 94.5% of its original amount after a year. What is the half-life of tritium-3? How long would it take for the sample to decay to 30%?

10. Match the graphs of the functions with the graphs of the derivatives.