

3. Suppose α is a real number, and that for all n there exist integers A_n and B_n such that $0 < A_n\alpha + B_n \leq \frac{1}{n}$. Show that α must be irrational. (15 points)

4. Prove that $\sqrt{52}$ is irrational. (15 points)

- Using straightedge and compass, construct a segment of length $\sqrt{3}$ given a segment of length 1. In addition, give the steps in the process. (15 points)

6. Consider the field $Q[\sqrt{3}]$, and the basis $e_1 = 1$ and $e_2 = 1 + \sqrt{3}$. What is the matrix for the element $2 + \sqrt{3}$ under this basis. Express the inverse of this element in terms of e_1 and e_2 . (15 points)

7. Outline the proof that you cannot construct $\sqrt[3]{2}$ with straightedge and compass. You should include all of the major steps, starting with the statement that the constructible numbers form a field. Thus you should explain what major results are needed to show this result, and not simply quote one theorem, but rather outline how that theorem is proven. (20 points)