

Math 496 Homework

Due Friday, Nov. 10.

1. Show that $\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}}$ is a rational number. Hint: Find rational numbers $a, b, c,$ and d such that $(a + b\sqrt{3})^3 = 26 + 15\sqrt{3}$ And $c + d\sqrt{3} = 26 - 15\sqrt{3}$.
2. Find a polynomial with rational coefficients that $\sqrt[3]{26 + 15\sqrt{3}}$ is a root of.
3. Solve with radicals by hand (showing your work) the equation

$$x^3 + 6x = 12.$$

4. Solve with radicals by hand (showing your work) the equation

$$x^3 + 3x^2 + 15x + 3 = 0.$$

5. It is unknown whether $e\pi$ and $e + \pi$ are transcendental or not. Curiously, it is known that at least one of them must be transcendental. Use the following outline to prove this:
 - (a) Show that either $e + \pi$ or $e - \pi$ is transcendental. (You are allowed to use that e and π are transcendental, and that the algebraic numbers form a field.)
 - (b) Calculate $(e + \pi)^2 - 4e\pi$ and factor.
 - (c) Using that the algebraic numbers are closed under square roots, show that if both $e + \pi$ and $e\pi$ are algebraic then $e - \pi$ would also be algebraic.
 - (d) Using the first part, show that either $e + \pi$ or $e\pi$ is transcendental.
6. Discuss the difference between transcendental and algebraic numbers and how this affects a student's ability to understand each. How might this influence your teaching about π and e ?