Disciplinary Styles in the Scholarship of Teaching and Learning

Exploring Common Ground

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Bridging the Divide

Research Versus Practice in Current Mathematics Teaching and Learning

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Within the discipline of mathematics, it is possible to distinguish two groups that have been concerned with the nature of education and its assessment. The most recognized group consists of mathematics education researchers who are addressing epistemological questions, building and testing educational theories, and conducting research into how students think and learn about mathematics. But in recent years, a second group has emerged, consisting of increasing numbers of teaching mathematicians who have been led to reexamine their individual efforts in ways that can lead to a more broadly based scholarship of teaching and learning. The story of the scholarship of teaching and learning in the mathematics profession can be witnessed through the creative tension that exists between the mathematics education researchers and the teaching mathematicians.

This essay first surveys research into undergraduate mathematics education and considers the various catalysts that have brought about recent changes. It then discusses the divide between mathematics education researchers and teaching mathematicians, going on to treat ways of bridging that divide through the scholarship of teaching and learning. It illustrates how new styles of scholarship in teaching and learning are reflected in the work of a particular group of Carnegie Scholars and considers problems of method and relevance, interdisciplinary activities, and sources of funding for research about mathematics education.
The World of Undergraduate Mathematics Education Research

Two major professional mathematics societies exist, the Mathematical Association of America (MAA) and the American Mathematical Society (AMS). A half-serious characterization of the difference between the two is that the MAA concentrates on teaching and research, while the AMS concentrates on research and teaching. The priority differences are nontrivial, but nearly 7,000 mathematicians are members of both. Each group has about 27,000 members at present. The two organizations hold joint annual meetings each January.

The coexistence of these two organizations says something about the level of interest of mathematics relative to the scholarship of teaching and learning. Few sciences can claim the same degree of interest in teaching and learning in their professional societies. In most cases, the research organization dominates, and a substantial percentage of members are employed outside the academy. Although some organizations in the various sciences are devoted to education, there is not the significant overlap in membership or support for joint activities that is present in mathematics. Most members of the two major mathematical organizations are employed in colleges and universities, leading to virtually unique opportunities for pursuing the scholarship of teaching and learning in the context of national professional meetings that involve a great number of the most active members of the profession employed in colleges and universities.

This said, scholarly inquiry into mathematics undergraduate education is in its infancy. "Mathematicians are used to measuring mathematical lineage in centuries, if not millennia; in contrast, the lineage of research in mathematics education (especially undergraduate mathematics education) is measured in decades" (2000: 649), puts Alan Schoenfeld, a mathematician with well respected credentials in the field of mathematics education research. He then goes on to point out that the journal Educational Studies in Mathematics dates to the 1960s and the Journal for Research in Mathematics Education to January 1970, while the first series focused on postsecondary education in mathematics, Research in Collegiate Mathematics Education, began to appear in
1994. Currently about a dozen journals exist whose primary purpose is the publication of articles related to teaching and learning of mathematics.

One of the great strengths of the mathematical community with respect to educational issues is the positive attitude of the major publications. Notices of the American Mathematical Society routinely includes articles that respond to teaching and learning issues in the public domain; for example, the participation of mathematicians in the design and implementation of curricular programs in elementary and secondary schools and in reform movements in college and university mathematics teaching. The American Mathematical Monthly of the MAA has the largest number of subscribers of any mathematical journal, and it is devoted to high-level mathematical exposition. Other MAA journals, Mathematics Magazine and the Collegiate Mathematics Journal, also stress expository style, and all three include articles on mathematical pedagogy. This inclusion represents a conscious decision on the part of the MAA editors to cover issues closely related to teaching and learning, especially those dealing with research into theory and practice in mathematics education.

Mathematics has been particularly fortunate in having the support of the National Science Foundation (NSF), which funds and manages a significant portion of the education research and curriculum reform in the field. In assessing the environment for the scholarship of teaching and learning, it is important to note that a number of nationally recognized mathematicians have given substantial time and energy to improve the undergraduate mathematics experience for their students. Unfortunately, many vocal mathematicians resent the funding that NSF has (in their words) "siphoned off" from pure research to support education initiatives. This same group would question the wisdom of those who have invested their effort to improve the teaching and learning of mathematics in lieu of pursuing other lines of mathematical research. A great deal of work remains to be done to create an environment in the mathematics community that fully supports and rewards the scholarship of teaching and learning in the same ways that it supports the scholarship of discovery.
Catalysts for Inquiry

Much of the contemporary research in undergraduate mathematics education finds its roots in the Calculus Reform Movement, advances in technology, or frequently in both phenomena. David Smith, codirector of Project CALC, one of the first NSF-funded calculus reform curricula, explains how the movement came into being:

It’s not hard to trace how we got out of touch with the needs of our students. Those of us educated in the Sputnik era were in the target population of that “traditional approach” — just at the end of a time when it didn’t matter much that the majority of college graduates (an elite subset of the population) didn’t know much about science or mathematics. As we became the next generation of faculty, the demographics of college-going students broadened significantly, new money flowed to support science, and broad understanding of science became much more important. The reward structure for faculty was significantly altered in the direction of research — away from teaching — just when we were confronted with masses of students whose sociology was quite different from our own. (1998: 780)

Smith goes on to describe some of the effects of the Sputnik era on educational practice in the United States:

This oversimplifies a complex story, but our response was to water down expectations of student performance, while continuing to teach in the only way we knew how. We created second-tier courses (e.g., calculus for business and life sciences), we wrote books that students were not expected to read, and we dropped test questions we didn’t dare ask. The goal for junior faculty was to become senior faculty so we wouldn’t have to deal with freshman courses. Along the way, we produced high-quality research and excellent research-oriented graduate students to follow in our footsteps. But seldom was there any opportunity or incentive to learn anything about learning — in particular, about how our students learn. (1998: 780)

Continuing his analysis, Smith then describes the genesis of the movement known as Calculus Reform. “In the mid-1980s there was widespread recognition that something was wrong. Calculus was chosen as the first target for ‘reform’ because it was both the capstone course for secondary education and the entry course for collegiate mathematics. Thus was born the ‘Calculus
Reform Movement" (Smith 1998: 777). The NSF responded to the call for calculus reform with a major funding initiative that produced a wide variety of calculus curricula.

What ensued as a result of the reform effort was a national conversation among mathematics educators regarding what students should be learning in the first two years of undergraduate mathematics. Of major note is that the early reformers challenged one another to think not only about what topics should be covered but also about effective ways to help students develop mastery and understanding of the topics. The discussion was wide-ranging and often heated, and brought with it the call for a more critical examination of what was happening in many mathematics classrooms across the country. The reform courses became more publicly scrutinized. Students were pretested, posttested, interviewed, and surveyed. Exam results were analyzed. The results were interesting and provocative but produced many more questions than answers, and teaching mathematicians began to recognize the challenge of investigating what was taking place in their own classrooms and to appreciate the work of their professional colleagues in the field of mathematics education research.

Schoenfeld, chair of the NSF Working Group on Assessment in Calculus, describes the broadening landscape in the introduction to Student Assessment in Calculus:

Following some years of development, it was appropriate to take stock — not so much for evaluation (deciding if efforts were successes or failures, in essence assigning a grade to the reform effort), but rather for assessment: to see what had been learned, to determine if what had been learned could be broadly applied, and to determine what steps might next be taken. Toward these goals NSF convened a Workshop on Assessment in Calculus Reform Efforts in July 1992. That workshop brought together educators and mathematicians who had been involved in the calculus reform movement and in understanding mathematics thinking and learning to discuss the state of the art. What were the reformers' goals? What had they tried, for what reasons? What had they learned? And what additional information was needed in order to make continued progress?

Two things became clear at that meeting. First, the issue was not limited to reform calculus: the issue was student learning in any calculus courses or courses related to them. The field needed better ways to examine student understanding, and all students and
faculty would profit from having access to such tools. Second, there was a clear need to develop a broad and coherent way of examining student understanding in calculus — to clarify the goals of instruction and to develop and refine ways of examining student behavior to help the mathematics community understand whether and how new forms of instruction are helping students attain those goals. (1997: 2)

Concurrent with the Calculus Reform Movement was the emergence of a professionally recognized research community focused on issues of undergraduate mathematics education. This community overlaps with, but is in no way coextensive with, the group that came together for calculus reform. The professional recognition of this community by the American Mathematical Society and Mathematical Association of America began in January 1991, with an organized paper session on research in undergraduate mathematics education. Three years later, the field saw the publication of the first volume of the AMS-MAA series Research in Collegiate Mathematics Education, in which some of the papers from the special session appeared. The community of researchers into collegiate mathematics education continued to grow and to coalesce. In January 1999, the Association for Research in Undergraduate Mathematics Education (ARUME) became the first special-interest group recognized by the MAA.

Over this same period of time, advances in technology were influencing the mathematics curriculum. From the graphing calculator to Web-based course delivery systems, new technologies have changed both the pedagogy and the content of mathematics courses throughout the undergraduate curriculum. These technologies continue to offer new and sophisticated calculation systems and dynamic visualization enhancements that the mathematics profession simply cannot ignore.

The response of mathematics educators to technology is somewhat different from that of most other fields. A chemist or physicist might use computer graphics to model the structure of a molecule or predict the results of a clash of galaxies. The simulations might indicate some relationships previously unnoticed, which can lead to refinement of the simulation process. But in mathematics, we are technically not simulating phenomena; rather, what we see truly are the phenomena we want to study,
whether simple arithmetic calculations, algebraic expressions, or geometric shapes. More and more, teachers are using technology to illustrate their lectures and provide motivation and applications. Students are able to work with many more examples than could be handled without computers, and they can experiment with mathematical phenomena that were just not accessible before. The big question, of interest to teachers and education researchers alike, is the role that technology plays in a mathematics curriculum for increasing the effectiveness of teaching and for enhancing students' learning.

While the mathematics community remains largely divided about any number of issues related to calculus reform and the use of advanced technology in the classroom, no one would contest that it has made a substantial impact on the profession. Nowhere has its impact been more greatly felt than in the mathematics education community. Out of this environment emerged a mathematics education research community more focused on what is happening in undergraduate collegiate mathematics and a mathematics teaching community more inclined to think about issues related to student learning. Calculus reform and advances in technology have served to expand, legitimize, and necessitate discussions regarding the teaching and learning of undergraduate mathematics.

**The Divide Between Education Researchers and Teaching Mathematicians**

Unfortunately, the issues concerning teaching and learning that are important to mathematics faculty are quite different from those that are important to mathematics education researchers. According to Schoenfeld, mathematicians and education researchers tend to have different views of the purposes and goals of research in mathematics education. "Research in mathematics education has two main purposes, one pure . . . to understand the nature of mathematical thinking, teaching, and learning; and one applied . . . to use such understandings to improve mathematics instruction. . . . [These dual purposes] contrast rather strongly with the single purpose of research in mathematics education as seen

To underscore this difference, we find in the literature that mathematics education researchers are most interested in questions about teaching and learning that can be redefined in theoretical terms. A sample list of such questions was generated at the Conference on Research in Collegiate Mathematics Education in 1996 and subsequently published in *Research in Collegiate Mathematics Education III*. The list includes questions such as “What is the nature of mathematical definitions? What is the epistemological status of examples in college mathematics texts? What is the nature of abstraction in the learning of mathematics? What is the nature of transfer? Does it exist? What is the nature of mathematical understanding?” (Selden and Selden 1998: 308-313).

We can contrast these more theoretical questions being posed by mathematics education researchers with the more practical ones being asked by mathematics educators. Many lists of such questions exist. The following sample was generated at the 1995 Oberwolfach Conference:

Do the tools of technology change students’ understanding of mathematics and, if so, how? . . . What are the difficulties that students have with formal mathematical language? . . . What pedagogical strategies can be effective in helping students understand the systematic development of mathematical theories? . . . What course designs and pedagogical strategies are most effective in taking into account the wide range of abilities and backgrounds of students? What are the pedagogical advantages and disadvantages of the different ways in which technology can be used? How does class size affect learning? . . . What group sizes in cooperative learning best support learning? What are the advantages and disadvantages of using applications from both inside and outside mathematics and of using history? (Kaput et al. 1996: 215-217)

The divide between the community of education researchers and teaching mathematicians extends beyond the issues and questions of interest to each group. Another issue at work here is that the mathematics education research community is very much rooted in the epistemology and methodology of psychology and education. Mathematicians as a whole are both skeptical and ignorant about these research paradigms. Lynn Steen, a mathematician who has been active in the mathematics education
reform community, muses that mathematicians rarely recognize education research as a valuable tool to confront problems in undergraduate mathematics education. In 1994, he published a list of questions that in many ways has helped define the challenge and frame the debate between education researchers and mathematicians. Steen’s list of questions also serves to illuminate the skepticism of the mathematics community toward the usefulness of research in mathematics education. At the top of the list was the question posed also by Schoenfeld:

Is the purpose of educational research to understand education or to improve it? This is, of course, the fundamental dichotomy between basic and applied research. The answer, perhaps, is “both.” But then one might ask a more difficult question: Are the insights from basic research (“understanding”) useful for applications (“improving”)? Does the transfer from theory to practice ever work in education? Can one hear the signal amid the noise? The linkage between teaching and learning is mediated by numerous factors whose variability is enormous and largely beyond the control of any researcher. In this environment, the observed effects of the few variables we can control are quite likely to be indistinguishable from the many we cannot control. Can we be sure that effects we observe are due to the causes we have created? Who is qualified to conduct research in undergraduate mathematics education? Are the results publishable? Will anyone read them? Does it count for tenure? (Steen 1994: 225-229)

Bridging the Divide Through the Scholarship of Teaching and Learning

The debate over the purpose and usefulness of research in mathematics education is leading to the emergence of a scholarship of teaching and learning in mathematics that may help bridge the divide between mathematics education researchers and teaching mathematicians. In particular, a growing number of teaching faculty are beginning to reflect more systematically on the quality of learning that may or may not be taking place. Most of their questions fall somewhere on the continuum of the epistemological issues being posed by mathematics education researchers and the more practical questions being asked by the mathematics community at large. The work of these individual faculty members is
more focused on issues important to their own teaching practices, and the forms that this work takes are deeply situated in the cultures of their own institutions. Much of the examination of their own courses has been carried out in private or shared with a few sympathetic colleagues; rarely does it become public. The sample sizes are usually too small and their methods of inquiry usually too informal to make their work of interest to mathematics education researchers or to other teaching mathematicians.

Nonetheless, some changes encourage these rank-and-file mathematics faculty to think of assessment strategies that address the quality of teaching and learning in their courses and their teaching projects. One force in this process is the National Science Foundation, the primary agency that supports mathematicians. The VIGRE (Vertical Integration of Research and Education) grants from the Division of Mathematical Sciences expect reports on the effectiveness of related education programs, and each proposal must include a discussion of the means of assessment. Similar provisions are in place in the NSF grant programs of the Division of Undergraduate Education and Education and Human Resources. The prospect of a grant to support efforts of teachers and students has focused the attention of mathematics teachers to a great extent on the assessment of their projects and to some extent on the related scholarship of teaching and learning.

Another movement that is encouraging teachers across all disciplines to examine their efforts in new ways is The Carnegie Academy for the Scholarship of Teaching and Learning (CASTL). In the case of mathematics, this effort has led participants in the CASTL program to reassess their work in terms of its aims, its transportability, and the degree to which assertions about the projects can be tested and validated. The work of Carnegie Scholars, while informed and strengthened by the literature in mathematics education, is clearly a different manifestation of the scholarship of teaching and learning. As examples, we mention the work of two Carnegie Scholars of mathematics.

Peter Alexander is at Heritage College, a small, independent, nonsectarian college on the Yakama Indian Nation reservation in south central Washington State. Alexander's project is attempting to define a "Sense of Numbers in Contexts" and to assess undergraduates' skill in this area. Alexander's long-term goal is to deter-
mine how people perceive and respond to numbers relating to social and citizenship issues. This goal is particularly important for the people in his rural, agricultural area, where 60 percent of the population is Mexican-American with large Native-American and white minorities. The amount of literature is growing on mathematical literacy and assessment of students’ knowledge of mathematics. The project will provide examples and evidence from a context that is not generally considered by mathematics educators.

Tom Banchoff has been teaching courses with interactive computer laboratories for more than 20 years. During the past six years, he and his assistants have developed Internet-based courses at Brown University, including a general education course called The Fourth Dimension and freshman honors courses in multivariable calculus and linear algebra. The newly developed software facilitates communication between students and instructors and among students, and computer graphics software makes it possible for instructors and students to create mathematical illustrations, demonstrations, and animations. As recent president of the MAA, Banchoff has given a large number of presentations on this work at meetings and conferences and at schools and colleges. The next phase of his projects involves an extensive assessment of the way such technology can improve the effectiveness of teaching and the quality of students’ learning.

Problems of Method and Relevance

Mathematicians, perhaps more than scholars in most other disciplines, struggle with the methods of research and the nature of results associated with research in mathematics education. *Heeding the Call for Change* contains an article that captures the spirit of this struggle.

By far the biggest bone of contention in the whole discussion of research in collegiate-level mathematics education is over the question of quality: Can the mathematics educational research establishment convince the mathematics community that its standards are up to snuff? . . . The dispute stems partly from fundamental differences between mathematical and educational research. The nature of the work is different, and the standards for one do not apply to the other. In mathematics, while research is
ultimately judged on its significance and depth, there is a first filter of logical correctness: Mathematicians prove theorems, and a proof is either valid or it isn't. . . . There's no corresponding abstract canon of rigor in educational research.

Instead, the criteria for quality in educational research have a more social cast to them. In particular, new research is judged in relation to previous studies—new work is expected to build on what others have found. . . . One of the main criteria is the presence of a theoretical framework. Another is the presence of an established methodology. These are admittedly fuzzy criteria, calling for considerable judgment on the part of people in the field. It is a paradigm that many mathematicians are clearly uncomfortable with, and some reject outright. (Cipra 1992: 169)

Schoenfeld echoed this sentiment a decade later in an article published in the Notices of the American Mathematical Society:

The nature of evidence and argument in mathematics education is quite unlike the nature of evidence and argument in mathematics. . . . When mathematicians use the terms "theory" and "models," they typically have very specific kinds of things in mind, both regarding the nature of those entities and the kinds of evidence used to make claims regarding them. . . . In mathematics theories are laid out explicitly. Results are obtained analytically: We prove that the objects in question have the properties we claim they have. . . . Models are understood to be approximations, but they are expected to be very precise approximations in deterministic form. . . . Descriptions are explicit, and the standard of correctness is mathematical proof. (2000: 641, 643)

This, Schoenfeld notes, contrasts starkly with the nature of argument in mathematics education research, where:

Findings are rarely definitive; they are usually suggestive. Evidence is not on the order of proof, but is cumulative, moving towards conclusions that can be considered to be beyond a reasonable doubt. A scientific approach is possible, but one must take care not to be scientific—what counts are not the trappings of science, such as the experimental method, but the use of careful reasoning and standards of evidence, employing a wide variety of methods appropriate for the tasks at hand. (2000: 649)

In making the case for mathematics faculty to be more open to the variety of methods used by educational researchers, Schoenfeld could just as easily be making the argument for educational researchers to be more open to the classroom-based
methods used by teaching faculty to investigate what is happening in their own courses. While the teaching faculty may lack sophisticated research methodologies, they bring to the field the practitioner's viewpoint, a deep level of understanding of how mathematics research is done, a knack for asking questions that appeal to other practitioners, and a desire to find applicable results. Moreover, they have shown a great willingness to go public with their findings, enriching the anecdotal data available for analysis.

**Interdisciplinary Horizons**

Evidence exists of a much greater level of interdisciplinary activity in mathematics than just a few years ago. In many ways, this change is being driven by the increasingly integrated nature of problems in mathematics and science. In the research area at Mathematical Challenges of the Twenty-First Century, a major conference held at UCLA in Summer 2000 under the auspices of the American Mathematical Society, a substantial number of the plenary lectures were directed toward applications of mathematics in science, particularly physics and biology. The NSF has encouraged interdisciplinary activity in several of its major funding initiatives in recent years. This is true in teaching as well as in research, where multiyear grants supported institution-wide programs for integrating mathematics and science education as well as for outreach to areas outside mathematics and science.

In these days when mathematics departments come under fire from unsatisfied clients, it is important to have some solid information about models for interaction between teachers in mathematics and those in other disciplines that require mathematics. Recently, the MAA committee studying mathematics in the first two years of college or university initiated a series of workshops involving mathematicians, mathematics educators, and their counterparts in one or more of the client disciplines, concentrating on the effectiveness of introductory courses in mathematics taken by their students. Appearing in many of the reports coming out of these workshops is the call from our client disciplines to expose introductory mathematics students to solving problems in which the mathematics is deeply embedded in a scientific application. Even more interesting to note is the attention
that the client disciplines would like mathematicians to give to pedagogical issues. This recent MAA initiative certainly opens new avenues for collaboration and study across disciplines in science and mathematics.

**Conclusion**

The challenge we face in the mathematics community is bridging the divide between what Michele Artigue, professor of mathematics at Université Paris VII Denis Diderot, calls “the world of research” in mathematics education and “the world of practice”:

Research carried out at the university level helps us to understand better the difficulties in learning that our students have to face, the surprising resistance to solutions of some of these difficulties, and the limits and dysfunction of our teaching practices. ... As mathematicians we are well aware that we can learn a great deal even from simplistic models, but we cannot expect them to give us the means to really control educational systems. So we have to be realistic in our expectations, careful with generalizations. This does not mean that the world of research and the world of practice have to live and develop as separate ones — far from it. ... Finding the ways of making research-based knowledge useful outside the educational communities and experimental environments where it develops cannot be left solely to the responsibilities of researchers. It is our common task. (Artigue 1999: 1384-1385)

No one knows this better or expresses it more eloquently than Schoenfeld, who for more than 10 years carefully studied what was occurring in his own problem-solving course:

I will state unequivocally that the course could not be as effective as it is were it not for the research, which revealed important issues for instruction. But I can be equally strong in stating the converse. The theory of mathematical competence that developed over those 10 years could not have emerged and been refined in the way it was were it not for the course, which served as both a source of and a test bed for basic theoretical ideas. Both theory and practice were better off for their close interaction. (1999: 11).

The future of the scholarship of teaching and learning in mathematics is very much in the hands of both the mathematics education researchers and the teaching mathematicians who have a sincere interest in learning more about what is taking place in
their classrooms. To prosper, these two groups must negotiate the differences that arise from both their questions and their methods of research.

The mathematics education researchers have taken a bold first step. They are going public in as many forums as possible. They are educating the mathematics community about their work, through articles in popular and widely read publications such as the Notices of the AMS. They are associating their organization within the broader association of the MAA. For their part, teaching mathematicians who have taken up the scholarship of teaching and learning are beginning to make their questions and answers public and available for their teaching colleagues to critique and build on. It is left to be seen whether, together, these scholars of teaching and learning and mathematics education researchers can not just bridge the current gaps but eventually eliminate the divide that separates their efforts.

Note

1. One can see the results of the NSF funding, for example, in many of the publications about calculus, which include Priming the Calculus Pump: Innovations and Resources (Tucker 1990); Student Research Projects in Calculus (Cohen et al. 1992); The Laboratory Approach to Teaching Calculus (Leinback et al. 1991); Assessing Calculus Reform Efforts (Leitzel and Tucker 1995); and numerous articles in UME Trends.

References


