

Final Study Guide MATH 111

The final will be cumulative. There will probably be a very slight emphasis on the material from the second half of the class. In terms of the material in the first half, please use the other study guide. As before, the final will have two parts, a “no calculator/no study sheet” portion and a “calculator/study sheet” allowed portion. For any partial credit on the exam, you will need to show work. Below are listed some particular items from the above sections to think about.

- Inverses of square matrices: you should be able to use your calculator to compute the inverse of a square matrix. You should be able to use an inverse matrix to solve a set of linear equations.
- Linear Programming: You should be able to graph inequalities with two variables. You should be able to set up linear programming problems and solve two-dimensional linear programming problems by hand using the graphical method. You should know the existence of solutions theorems and be able to apply them in contexts. You should be able to identify when a solution set is bounded, and know how this fits together with the existence theorems. Given an optimization problem you should be able to reinterpret the problem as an optimization problem on inequalities. You should be able to solve these by hands when they involve only two variables. You should be able to set them up however, even if there are more than two variables.
- Simplex Method: You should be able to set up a simplex tableau given an inequality and/or a word problem. You should also know if a tableau is in final form, and to be able to interpret the results if it is.
- Financial Mathematics: You should be able to do the problems of financial mathematics. You need to be able to solve simple and compound interest problems, convert present and future value of money. You should also be able to calculate the future value of an annuity. You should also be able to find the present value of an annuity. You should be able to use the annuity and present/future value formulas to calculate a payment on a lump sum load and/or the current value of a lump sum loan. You should be able to use the annuity and present/future

value formulas to find the payment of an amortized loan. In addition you need to be able to solve problems involving loans, annuities, present, and future values.

- Precalculus: You should be able to do basic computations involving exponents, polynomials, rational functions (algebra of polynomials). You should also be able to solve problems involving inequalities and absolute values. You should know the exponent rules, how to use them, and be able to simplify algebraic expressions.
- Functions and their graphs: You need to know what a function is, be able to identify whether rules correspond to functions, and be able to determine the domain and range of a function. You should also be able to identify whether a graph is the graph of a function by the vertical line test. You should also be able to identify how points on the graph correspond to a function, and figure out information about the function from the graph.
- Algebra of Functions: You should be able to define/describe and calculate the sum, difference, product, and quotient of two functions. You should be able to find the domain of such functions. You should also be able to work with composition of functions, including finding a formula for a composition of two functions given their formulas. You should also be able to interpret the meaning of products, sums, quotients, differences, and compositions of functions in a context. Moreover, you should be able to come up with such examples yourself.
- You should know the definition of exponential functions. Understand the relationship of exponential functions to loans. You should be able to use the laws of exponents (p. 812) in problems to reduce expressions. You should also know the properties of the exponential function (p. 813). Moreover, you should know what continuously compounding interest is, how to calculate the annual interest given continuously compounding interest, as well as be able to calculate future value of objects based on inflation and continuously compounding inflationary value. You should also be able to do word problems that relate to exponential functions.
- Logarithmic Functions: You should know the definition of logarithmic functions and be able to solve problems involving logarithmic functions.

You should be able to relate logarithmic functions to problems involving interest and loans. You should understand the natural logarithmic functions and working with that function. You should be able to answer word problems that involve the logarithmic functions.

On the following sheets, you will see some sample problems (for these sections only), which are **not** meant to be a complete list of all types of problems on these chapters that will be on the final.

Sample Problems on
Algebra, Functions, Exponents, & Logarithms

Math 111

Part 1: No calculator or study sheet. Remember to get full credit, you must show your work.

1. Determine graphically the solution set for each system of inequalities and indicate whether the solution set is bounded or unbounded:

$$\begin{aligned}x - y &< 7 \\3x + 2y &\geq 6 \\x &\geq 0\end{aligned}$$

2. Suppose we are given a linear programming problem with a feasible set S and an objective function $P = ax + by$. Finish the following sentences:
 - (a) If S is _____ then P has both a maximum and a minimum value on S .
 - (b) If S is _____ and both a and b are nonnegative, then P has a minimum value on S provided that the constraints defining S include the inequalities $x \geq 0$ and $y \geq 0$.
 - (c) If S is _____ then the linear programming problem has no solution; that is P has neither a maximum nor a minimum value.
3. Give an example of a simplex tableau that is in final form and one that is not in final form.
4. Suppose that you take out a loan for \$30,000 over a 10 year term at 5% interest with monthly payments. At the end of 3 years do you anticipate that you would owe closer to \$15,000, \$17,500, \$20,000, or \$22,500. Explain. (You should think about how to solve this without a calculator.)
5. True or false: the future value of an annuity can be found by adding together all the payments that are paid into the account.

6. State the quadratic formula.
7. Factor out the greatest common factor from each expression
- (a) $3x^5 - 12x^3 + 9x^2$
 - (b) $4x^{2/3}y - 3xy^{1/3}$.
8. Factor $9a^2 - x^2$.
9. Given that 2 is a real root of the polynomial $f(x)$, state a factor of $f(x)$.
10. Evaluate the given expression
- (a) $|-5 + 3|$
 - (b) $\sqrt{3} \cdot |-4| + 2 \cdot |-\sqrt{3}|$.
11. Simplify the following expressions:
- (a) $\frac{x^{-1}+2x^{-2}}{x^2-4}$
 - (b) $\frac{-xe^x}{x+1} + e^x$
 - (c) $|\sqrt{2} - \sqrt{6}|$.
12. Find the values of x that satisfy the inequality (inequalities):
- (a) $-3x \leq 24$.
 - (b) $x^2 - 5x + 5 > 1$.
 - (c) $|x - 4| \leq 9$.
 - (d) $|3x + 7| \geq 5$.
13. Evaluate the following:
- (a) $(64)^4/3$.
 - (b) $\left[\left(\frac{-1}{3}\right)^2\right]^{-3}$.
14. Simplify
- (a) $(2x^3)(-4x^{-2})$.

(b) $\frac{e^x e^{4x}}{e^{2x-2}}$.

15. Solve for x : $10^{2x-1} = 10^{x+3}$.

16. Use the logarithm rules to simplify the given equations:

(a) $\ln(x(x+1)(x+2))$.

(b) $\ln(x^{x^2+1})$.

17. Express the given equation in logarithm form.

(a) $5^{-3} = \frac{1}{125}$.

(b) $e^{5x} = 72$.

Calculator and Study Sheet Allowed

1. Graph the functions on the same axis:

$$y = e^{3x}$$

$$y = \ln(3x).$$

2. Graph the functions on the same axis: $y = 2^x$, $y = 3^x$, and $y = 4^x$. Clearly label each.
3. The revenue of Leisure Travel is given by $f(x)$ dollars, where x is the dollar amount spent by the company on advertising. The amount spent by Leisure at time t on advertising is given by $g(t)$ dollars. What does the function $f \circ g$ represent?
4. Investment A offers a 10% return compounded semiannually, and investment B offers a 9.75% return compounded continuously. Which investment has a higher rate of return over a 4-yr period?
5. Find the interest rate needed for an investment of \$5000 to grow to the amount of \$7500 in 3 years if interest is compounded monthly.
6. How long will it take an investment of \$4000 to triple in 5 years if the investment earns interest at the rate of 8% compounded quarterly. Approximately how long will it take to double?
7. Suppose you take out a home loan of \$400,000 at 7% interest amortized over 30 years. The mortgage company allows you to pay no interest for the first year of the loan. At the end of the first year, the loan converts to a traditional mortgage (with only 29 years left). How much are the payments at the end of the first year? How does this compare to what the payment would be if you had taken a traditional 6.9% 30 year loan (with monthly payments).
8. Suppose tuition at Georgetown is increasing at 12% per year, and currently costs \$20,000 per year. How much will tuition at Georgetown be in 5 years?

9. A T-shirt company wants to manufacture 2 types of T-shirts. The first T-shirt requires 10 minutes on machine *A*, 5 minutes on machine *B*, and 3 minutes on machine *C*. The second T-shirt requires 7 minutes of manufacturing time on machine *A*, 6 minutes of manufacturing time on machine *B*, and 4 minutes of manufacturing time on machine *C*. The first T-shirt sells for a profit of 5 dollars and the second T-shirt sells for a profit of 6 dollars. Set up the linear programming problem for this company. Label all of your variables.
10. Set up a simplex tableau associated to the programming problem

$$\begin{aligned}
 P &= 4x + 5y + 2z \\
 3x + 5y - z &\leq 100 \\
 2x + 7z &\leq 125 \\
 4y + z &\leq 110 \\
 x, y, z &\geq 0
 \end{aligned}$$

Set up a simplex tableau for this problem.

11. Suppose a linear programming problem has the following set up: x denotes the number of widgets to be produced, y denotes the number of thingymabobs to be produced, and z denotes the number of whatchamacallits to be produced. There are 3 machines that each product requires time on. Suppose the final tableau for this problem is:

$$\left[\begin{array}{ccccccc|c}
 x & y & z & u & v & w & P & \\
 0 & 1 & 0 & 6 & 0 & 2 & 0 & 10 \\
 0 & 0 & 2 & 3 & 1 & 8 & 0 & 20 \\
 1 & 0 & -3 & -4 & 0 & -3 & 0 & 15 \\
 \hline
 0 & 0 & 3 & 8 & 0 & 4 & 1 & 100
 \end{array} \right]$$

What is the final solution associated with this tableau? Interpret this solution using words. Does any machine have slack time?

12. Determine the monthly payment that would be made on a 5 year car loan for \$40,000 car at 7.5% annual interest payable every month.
13. Price publishing sells encyclopedias under two payment plans: cash or installment. Under the installment plan, the customer pays \$22 per month over a 3-year period with interest charged on the balance at a

rate of 18% per year compounded monthly. Find the cash price for a set of encyclopedias if it is equivalent to the price paid by a customer using the installment plan.