Study Guide 1
MATH 111

The test will cover the material from Chapter 1 and up through Section 3.2. There will be two sections of the test, a no calculator or study sheet portion and a calculator and study sheet allowed portion and a calculator portion. For the second section you may have a study sheet of one 8 1/2 by 11 sheet of paper with whatever material you want written down on it. I highly recommend that you write up and organize your study sheet yourself. Most of the questions on the test will be similar to homework problems (and **any** homework problem is fair game) or problems on this study guide. On all problems you must show your work to receive full credit. You also will need to make clear what your answer for a given problem is. Particular topics you should know are:

- Straight lines - you should know the three forms of equation for a straight line and be able to convert from one form to another. You should be able to calculate slopes of lines given points.

- Linear Functions and Mathematical Models - you should be able to model appropriate situations with lines. In particular, you will need to be able to write down a cost function, a revenue function, and a profit function. In addition, you should be able to model supply and demand situations with lines.

- Intersection of straight lines - you should be able to find the intersection points of two straight lines. In addition, you should know the different possibilities for the intersections of straight lines. You should be able to apply these ideas to models to find the break-even level, to make decision analyses, and to find market equilibriums.

- Solutions of systems of equations - You should be able to work with systems of equations, to recognize linear equations and to solve word problems involving systems of equations.

- Gauss-Jordan method of solving linear equations - you should be able to carry out individual steps of the Gauss-Jordan method for solving linear equations. You should be able to deduce a correct next step for the method, and you should be able to carry it out in simple situations.

Most importantly, you should be able to interpret an augmented matrix as a set of equations, and be able to deduce information from the row reduced echelon form of the matrix.

- Over- and Under-determined systems of linear equations - you should know what an over-determined and an under-determined system of linear equations are. You should know how to tell whether a system is over- or under-determined from the row reduced echelon form of the matrix associated to the system. You should also know the theorem about what the possibilities are for the number of solutions of a system of equations in terms of the number of variables and the number of equations.

- You should know basic matrix notation and algebra (including addition of matrices, multiplication of a matrix by a scalar, and the multiplication of matrices). You also need to know the basic notation of matrices, in terms of the ith row and jth column, etc. You should be able to use matrices to represent data.

- Inverses of square matrices: you should be able to use your calculator to compute the inverse of a square matrix. You should be able to use an inverse matrix to solve a set of linear equations.

- Linear Programming: You should be able to graph inequalities with two variables. You should be able to set up linear programming problems and solve two-dimensional linear programming problems by hand. You should know the existence of solutions theorems and be able to apply them in contexts. You should be able to identify when a solution set is bounded, and know how this fits together with the existence theorems. Given an optimization problem you should be able to reinterpret the problem as an optimization problem on inequalities. You should be able to set them up.

On the following sheets, you will see some sample problems.

Sample Problems for Test 1
Math 111
Part 1: No calculator or study sheet

1. Find the slope of a line through the points $(2, -3)$ and $(5, 1)$.

2. List all possibilities for the number of solutions that a system of 3 equations in 3 unknowns can have. Does this answer change (and if so how) if we have 3 equations in 4 unknowns?

3. Write the augmented matrix associated to the equations

$$\begin{aligned} 2x + 4y - z &= 1 \\ 3x - 2y + z &= 2 \\ 5x - y &= -2 \end{aligned}$$

4. If a graph has a negative slope, as you move to the right on the graph, do the y-values get larger or smaller?

5. If

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

   is the reduced row echelon form of the matrix A associated to a system of equations (in x, y, and z), write the solution of the corresponding system of equations.

6. What are the three different forms for the equation of a line.

7. Write down a matrix in row-reduced form for which the corresponding system of equations has **NO** solutions.

8. Write down a matrix in row-reduced form for which the corresponding system of equations has infinitely many solutions.

9. If A is a 3×3 matrix, and

$$v = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix},$$

What is $v + A^{-1}Av$?

10. Are the two matrices below inverses? Explain how you know.

$$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

11. Determine graphically the solution set for each system of inequalities and indicate whether the solution set is bounded or unbounded:

$$\begin{aligned} x - y &< 7 \\ 3x + 2y &\geq 6 \\ x &\geq 0 \end{aligned}$$

12. True or False (Circle the letter). Partial credit will be given only if an explanation is included.

T **F** The matrix $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 0 & 0 \end{array}\right]$ corresponds to a system of equations with no solutions.

T **F** The matrix $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right]$ corresponds to a system of equations with exactly one solution.

T **F** The matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 7 \end{array}\right]$ is in row-reduced echelon form.

T **F** The matrix $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$ is in row-reduced echelon form.

T **F** If A is an $m \times n$ matrix and B is an $m \times k$ matrix, then AB is an $n \times k$ matrix.

13. If $A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$, find the product matrix AB.

Calculator and Study Sheet Allowed

14. Write the equation of the line passing through the points $(3, 5)$, and $(-3, 17)$.

15. What is the slope of the line $3x + 5y = 9$?

16. A manufacturer has monthly fixed costs of $2,600$ and a production cost of \$2 for each unit produced. If the product sells for 5 per unit, write

    (a) The cost function $C(x)$.

    (b) The Revenue Function $R(x)$.

    (c) The Profit Function $P(x)$.

    (d) What is the break-even point for this system?

    (e) The Profit at the break-even point is.

17. Use any method you like to find the point of intersection of the lines $2x + 5y = -1$ and $3x + y = -8$.

18. Consider the system

$$\begin{aligned} 2x - 4y &= -6 \\ -3x + 6y &= k. \end{aligned}$$

For what values of k does the system have

(a) no solutions?

(b) exactly one solution?

(c) infinitely many solutions?

19. Write the augmented matrix for the system

$$\begin{aligned} 2x - z &= 1 \\ 3x + 2y &= 4. \end{aligned}$$

Be careful!!

20. For what values of k, m, and n is $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ k & m & n & 3 \end{array}\right]$ in row reduced form?

21. Use the indicated row operations to fill in the 5 missing entries in the second matrix below:

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & -4 \\ 0 & -4 & -1 & -6 \end{array}\right] \begin{array}{c} \\ R_1 - 2R_2 \\ \longrightarrow \\ R_3 + 4R_2 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & & -5 & \\ 0 & 1 & 3 & -4 \\ 0 & & & \end{array}\right]$$

22. **(a)** Write down the augmented matrix for

$$\begin{aligned} x - z &= -2 \\ y + 2z &= 7 \\ -x + 2z &= 6. \end{aligned}$$

(b) Write down what operation should be the first row operation to undertake in Gauss-Jordan elimination.

23. Sandy has a total of \$3,500 on deposit with two savings institutions. One pays interest at the rate of 6% per year and the other pays interest at the rate of 7% per year. If Sandy earned a total of \$250 in interest during a single year, how much does she have on deposit in each institution?

24. Below is a row-reduced matrix. Write down the system of equations corresponding to this matrix, and then write the solution for the corresponding equations using the parameter t.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

25. If $2\begin{bmatrix} x & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 3y & 2 \\ 0 & -8 \end{bmatrix}$, find x and y.

26. Write down the corresponding matrix equation to the system of linear equations below, and solve it (via calculator) using the inverse matrix (show the inverse matrix):

$$\begin{aligned} 2x + 3y + 4z &= 12 \\ x - 5y + 2z &= 4 \\ 3x - y + z &= 1 \end{aligned}$$

27. A T-shirt company wants to manufacture 2 types of T-shirts. The first T-shirt requires 10 minutes on machine A, 5 minutes on machine B, and 3 minutes on machine C. The second T-shirt requires 7 minutes of manufacturing time on machine A, 6 minutes of manufacturing time on machine B, and 4 minutes of manufacturing time on machine C. The first T-shirt sells for a profit of 5 dollars and the second T-shirt sells for a profit of 6 dollars. Set up the linear programming problem for this company. Label all of your variables.