1 Problem 1 (Chapter 12, #5)

1.1 Part a)

The opportunity cost of labor is a constant $w$. Since labor is the only input to production, the labor cost curve is a straight line with slope $w$. The cost of each unit of labor supplied is $w$. Production is just a standard concave function. To find the point of maximum surplus, we just want the level of labor at which the slope of the tangent line to the production curve is equal to the slope of the labor line. So we want the point on the production curve where the slope = $w$. On the picture below, $L^*$ is the point at which the surplus is maximized.
Note that it’s simple to show this without a graph, too. If $F(L_T)$ is the production function, then the surplus is given by

$$Surplus = F(L_T) - wL_T$$

To maximize the surplus, take the derivative with respect to $L_T$ and set equal to zero. This gives:

$$F'(L_T) = w,$$

which is exactly what the picture shows.

### 1.2 Part b)

The farmer wants to maximize her income. Call one farmer $A$ and the other one $B$. Start by writing down the farmer’s net income (earnings minus cost):

$$\frac{F(L_A + L_B)}{2} - wL_A$$

The farmer chooses $L_A$ to maximize this expression. So take the derivative:

$$F'(L_A + L_B) = 2w$$

Since there is a diminishing marginal product of labor this means that the total labor used on the farm $L_T = L_A + L_B$ must be less than in part a). The intuition is that the laborer does not reap the full benefit of increasing her labor and so less than the efficient amount of labor (which we solved for in part a) will be supplied.

### 2 Problem 2 (Chapter 13, #4)

#### 2.1 Part a)

We showed this in class. With a large supply of labor, many laborers are willing to work at the minimum piece rate needed to ensure that their income (in terms of food) is sufficient to make it possible to produce at that level of exertion. This is exactly the piece rate that is tangent to the capacity curve.

#### 2.2 Part b)

When workers had some nonlabor income, we saw that the picture looked like
curve shifted the capacity curve from the solid curve to the dotted one. The new minimum piece rate that is necessary to sustain an equilibrium with high production now falls because workers can use the nonlabor income to help them buy food. This piece rate is depicted by the dotted line that is tangent to the new capacity curve.

The key here is that there is a high supply of labor. So in the case without nonlabor income, there was a large group of people who would like to work at the minimum piece rate. Now that workers have a small amount of nonlabor income the minimum piece rate falls. Given that there is a large supply of laborers willing to work for the minimum piece rate, the equilibrium wage will simply fall to the new minimum piece rate.

You can also graph this in terms of labor supply and labor demand. You can see that the new equilibrium piece rate is lower. This only happens for sure in the case where supply is large so that the demand curve crosses the supply curve in the flat section. Intuitively, when supply is relatively large, the equilibrium piece rate will be the minimum, which is lower when there is nonlabor income.

Be clear that this logic applies only to the case of small amounts of nonlabor income.
2.3 Part c)

To show this, we need to show that the decrease in payments to labor are bigger than the increase in nonlabor income. This must be the case. Notice first that the whole capacity curve shifts the same horizontal distance. It’s a little hard to see on this picture, but the tangent point also moves down along the capacity curve. So labor effort is slightly lower in the case with nonlabor income. This fact adds to the fall in labor income beyond the shift in the capacity curve itself. Therefore, the fall in labor income has to be greater than the increase in nonlabor income.