1 Chapter 10, Problem 14

Define the utility of person $A$ to be $u_A(y_A)$ if her income is $y_A$. Define person $B$'s utility to be $u_B(y_B)$. Also define $y_{A1}$ and $y_{B1}$ to be each person's income if output is 1000 and $y_{A2}$ and $y_{B2}$ to be each person's income if output is 2000. To find an efficient rule, we have to maximize person $A$’s utility given whatever person $B$’s utility is.

Since person $B$ is risk-neutral, we can express her expected utility as:

$$ Eu_B = E y_B = \frac{1}{2} y_{B1} + \frac{1}{2} y_{B2} $$

Take person $B$'s utility to be $\frac{\pi}{2}$. Then:

$$ \frac{1}{2} y_{B1} + \frac{1}{2} y_{B2} = \frac{\pi}{2} \Rightarrow y_{B2} = \pi - y_{B1} $$

Notice that $y_{A1} = 1000 - y_{B1}$ and $y_{A2} = 2000 - y_{B2}$. Then $y_{A2} = 2000 + y_{B1} - \pi$. So we can express person $A$’s expected utility as:

$$ Eu_A = \frac{1}{2} u(y_{A1}) + \frac{1}{2} u(y_{A2}) $$

$$ = \frac{1}{2} u(1000 - y_{B1}) + \frac{1}{2} u(2000 + y_{B1} - \pi) $$

Now maximize person $A$’s utility with respect to the choice of $y_{B1}$. Taking the derivative with respect to $y_{B1}$ and setting equal to zero gives:

$$ -\frac{1}{2} u'(1000 - y_{B1}) + \frac{1}{2} u'(2000 + y_{B1} - \pi) = 0 \Rightarrow $$

$$ u'(1000 - y_{B1}) = u'(2000 + y_{B1} - \pi) $$
Since person $A$ is risk-averse, the derivative is decreasing everywhere and so the above condition implies:

$$1000 - y_{B1} = 2000 + y_{B1} - \pi$$

This is equivalent to:

$$y_{A1} = y_{A2},$$

which was what we wanted to prove.