Problem Set 2 Solutions

Economic Development

September 24, 2009

1 Chapter 10, Problem 14

Define the utility of person A to be $u_A(y_A)$ if her income is y_A. Define person B's utility to be $u_B(y_B)$. Also define y_{A1} and y_{B1} to be each person's income if output is 1000 and y_{A2} and y_{B2} to be each person's income if output is 2000. To find an efficient rule, we have to maximize person A's utility given whatever person B's utility is.

Since person B is risk-neutral, we can express her expected utility as:

$$Eu_B = Ey_B = \frac{1}{2}y_{B1} + \frac{1}{2}y_{B2}$$

Take person B's utility to be $\frac{\overline{u}}{2}$. Then:

$$\frac{1}{2}y_{B1} + \frac{1}{2}y_{B2} = \frac{\overline{u}}{2} \Rightarrow y_{B2} = \overline{u} - y_{B1}$$

Notice that $y_{A1} = 1000 - y_{B1}$ and $y_{A2} = 2000 - y_{B2}$. Then $y_{A2} = 2000 + y_{B1} - \overline{u}$. So we can express person A's expected utility as:

$$\begin{aligned} Eu_A &= \frac{1}{2}u(y_{A1}) + \frac{1}{2}u(y_{A2}) \\ &= \frac{1}{2}u(1000 - y_{B1}) + \frac{1}{2}u(2000 + y_{B1} - \overline{u}) \end{aligned}$$

Now maximize person A's utility with respect to the choice of y_{B1}. Taking the derivative with respect to y_{B1} and setting equal to zero gives:

$$\begin{aligned} -\frac{1}{2}u'(1000 - y_{B1}) + \frac{1}{2}u'(2000 + y_{B1} - \overline{u}) &= 0 \Rightarrow \\ u'(1000 - y_{B1}) &= u'(2000 + y_{B1} - \overline{u}) \end{aligned}$$

Since person A is risk-averse, the derivative is decreasing everywhere and so the above condition implies:

$$1000 - y_{B1} = 2000 + y_{B1} - \overline{u}$$

This is equivalent to:

$$y_{A1} = y_{A2},$$

which was what we wanted to prove.