1. For each of the following functions, determine whether it is one-to-one and whether it is onto.

   (a) $T : \mathbb{R} \to \mathbb{R}$ defined by $T(x) = x^3$.

   (b) Let $V = \{\text{all people who have ever lived}\}$ and $W = \{\text{all women who have ever lived}\}$. Define a function $T : V \to W$ by $T(x) =$ mother of x. (Ignore Adam and Eve.)

   (c) $T : M_{2\times 2} \to \mathbb{R}$ defined by $T(A) = \det(A)$.

2. In class we talked about the notion of *isomorphic* vector spaces. Recall that two vector spaces V and W are **isomorphic**, and we write $V \cong W$, if there is a one-to-one and onto linear transformation $T : V \to W$. In this problem we will show that various vector spaces are isomorphic.

   (a) Let $V = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \text{ are real numbers} \right\}$. We will show that $V \cong \mathbb{R}^2$.

   We begin by defining a function:

   $$T : V \to \mathbb{R}^2$$

   by

   $$T\left(\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \right) = \begin{bmatrix} x \\ y \end{bmatrix}$$

   Now you must show that T is (1) a linear transformation, (2) one-to-one, and (3) onto.

   (b) Show that $M_{2\times 2} \cong \mathbb{R}^4$. Define a function

   $$T : M_{2\times 2} \to \mathbb{R}^4$$

   (Look in your classnotes for help.) Then you must show that T is (1) a linear transformation, (2) one-to-one, and (3) onto.

   (c) Show that $P_3 \cong \mathbb{R}^4$. Define a function

   $$T : P_3 \to \mathbb{R}^4$$

   (Look in your classnotes for help.) Then you must show that T is (1) a linear transformation, (2) one-to-one, and (3) onto.

   (d) Your work in parts (b) and (c) show that

   $$M_{2\times 2} \cong P_3 \cong \mathbb{R}^4$$

   These problems just illustrate the (very big) fact we mentioned in class: If $\dim(V) = n$, then $V \cong \mathbb{R}^n$. So basically, every finite-dimensional vector space is just $\mathbb{R}^n$!!!